The Private Capital Alpha

Gregory Brown[*] Andrei S. Gonçalves[†] Wendy Hu[‡]

This Version: September 2024[§]

Abstract

The alpha of an investment reflects its ability to increase the Sharpe ratio of a benchmark portfolio allocation based on tradable factors. We argue that, in the context of private capital, the usual approach to estimate alpha is misleading because it ignores the economic realities of investing in private markets. We then combine a large sample of 5,028 U.S. buyout, venture capital, and real estate funds from 1987 to 2022 to estimate the alphas of private capital asset classes under realistic simulations that account for the illiquidity and underdiversification in private markets as well as the portfolio allocation of typical limited partners. We find that buyout as an asset class provided a positive and statistically significant alpha during our sample period. In contrast, over our sample period, the venture capital alpha was large and positive but statistically unreliable whereas the real estate alpha was very close to zero.

JEL Classification: G10; G11; G12; G20; G23; G24.

Keywords: Private Capital; Private Equity, Venture Capital; Real Estate; Performance Evaluation.

[*]University of North Carolina at Chapel Hill, Kenan-Flagler Business School and Institute for Private Capital. E-mail: gregwbrown@unc.edu.

[†]Fisher College of Business, The Ohio State University. E-mail: goncalves.11@osu.edu.

[‡]MSCI. E-mail: wendy.hu@msci.com.

[§]We are very grateful to MSCI and the Private Equity Research Consortium (PERC) for their support and access to the data used in this analysis. This paper benefited a lot from conversations with members of these institutions as well as from helpful comments from seminar participants at the 2023 IPC & PERC Private Equity Research Symposium.

Introduction

The allocation of institutional investors to private capital has drastically grown over the last decades. For instance, the aggregate target allocation of public pension funds to private capital has increased from around 8% in 2001 to around 22% in 2021 (Begenau, Liang, and Siriwardane (2023)). As such, measuring the risk-adjusted performance of private capital investments is extremely important. The usual approach in the literature is to estimate the Net Present Value (NPV) of private capital funds from the perspective of a Limited Partner (LP) who can freely allocate capital in public markets (see Kaplan and Schoar (2005) and Korteweg and Nagel (2016, 2024)). In contrast, we estimate the alpha of private capital asset classes from the perspective of a LP with realistic public market allocation. We also account for other economic realities of investing in private capital such as the effect of illiquidity and underdiversification on the allocation of LPs to private markets.

Our empirical analysis is based on the MSCI Private Capital (previously Burgiss) Universe dataset. We cover 5,028 U.S. Buyout (BO), Venture Capital (VC), and Real Estate (RE) private equity funds with vintage years from 1987 to 2022, representing $3.64 trillion in total fund commitments. We estimate the alpha of each of these private capital asset classes separately as well as the alpha of a value-weighted portfolio on these three asset classes combined (which we refer to as ALL).

Our main findings are as follows. First, the annualized alpha of BO over our sample period was positive (2.1%) and statistically significant (the 95% confidence interval covers from 0.0% to 4.0%). Second, while VC had a large annualized alpha over our sample (3.0%), the substantial uncertainty associated with VC investments makes this alpha statistically insignificant (the 95% confidence interval covers from -5.4% to 13.6%). And third, RE provided no alpha over our sample period (the RE alpha was -0.7% but statistically insignificant). Moreover, the ALL strategy provided an annualized alpha of 2.0% that is close to statistically significant (the 95% confidence interval covers from -0.5% to 6.8%).[1]

[1] The similarity of the BO and ALL results is due to the size of the total commitments to the private capital asset classes we study. In particular, the total commitment to BO over our sample period was $2.23

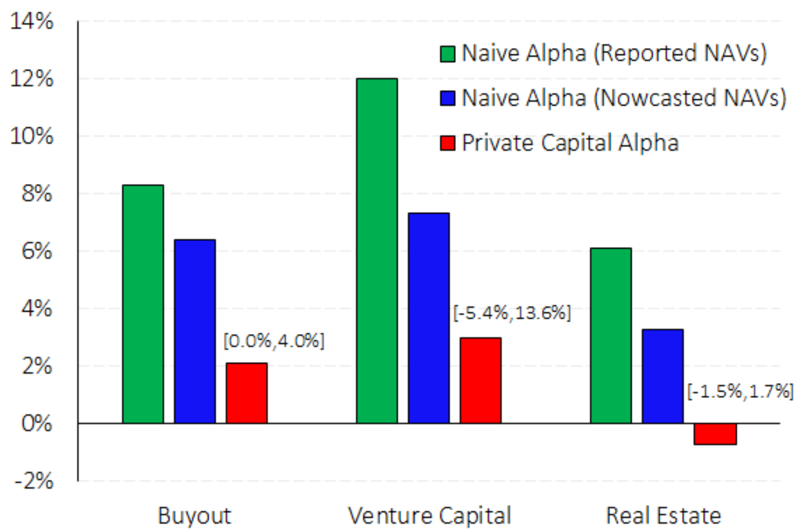


Figure 1
Main Result: The Private Capital Alpha

This figure provides the alphas of the private capital asset classes we study. The green and blue bars provide naive alphas based on regressions of private capital value-weighted indices onto equity and bond public market indices. The green bar computes private capital returns using reported NAVs whereas the blue bars rely on the nowcasted NAVs of Brown, Ghysels, and Gredil (2023) to account of NAV smoothing. The red bars provide alphas based on simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (with the median alpha used for the bars and the 95% confidence interval reported on the top of each bar). These alphas reflect the Sharpe ratio increase of a realistic portfolio that includes public and private investments relative to a baseline public market portfolio, normalized appropriately to be in alpha units.

A naive alpha estimation leads to very different results. To highlight this issue, we compute fund-level returns using reported cash flows and Net Asset Values (NAVs). We then aggregate fund-level returns into asset class returns as standard value-weighted indices. Finally, we estimate the alpha of each of these indices from the intercept of a regression of the respective index onto bond and equity public market indices. The results are provided in the green bars of Figure 1. The alphas are massive, with the lowest annualized alpha being a little higher than 6%. We refer to these values as "naive alphas" because they do not account for the

trillion whereas the total commitment to VC and RE combined was only $1.41 trillion over our sample.

economic realities of investing in private capital (detailed below).

One important economic reality of private capital investments is the smoothness of reported NAVs, which leads to understated risk and overstated risk-adjusted performance (see Getmansky, Lo, and Makarov (2004) and Couts, Gonçalves, and Rossi (2024)). To address this issue, we consider an alternative naive alpha estimation that uses the nowcasted NAVs of Brown, Ghysels, and Gredil (2023) when computing fund-level returns, but is otherwise based on the same procedure described in the above paragraph. The results for this alternative naive alpha are provided in the blue bars of Figure 1. While alphas decrease after using nowcasted NAVs, the annualized alphas are still large. For instance, the BO and VC alphas are still higher than 6%. We argue that these are still "naive alphas" as they do not account for the economic realities of investing in private capital beyond NAV smoothing.

These naive alphas have two main limitations. First, the naive alphas are based on private capital indices that include all funds of each vintage year whereas evaluation and monitoring costs would limit the number of funds a realistic LP would invest in annually. That is, realistic private capital investments are underdiversified whereas the indices underlying naive alphas are well diversified. Second, the naive alphas implicitly reflect the Sharpe ratio increase in the maximum Sharpe ratio portfolio in public markets to the maximum Sharpe ratio portfolio that combines public and private market investments. A realistic LP would not hold the maximum Sharpe ratio portfolio of public market indices in the absence of private capital opportunities (due to mandates, constraints, and informational frictions). Moreover, a realistic LP controls the capital commitments to private capital, but not the actual weight allocated to private capital since capital calls and distributions are under the control of the General Partners (GPs). As such, a realistic LP can never reach a pre-specified and fixed allocation to private capital, let alone the maximum Sharpe ratio allocation that combines public and private market investments.

To address these limitations, our private capital alpha is based on simulations of realistic investments in private capital. Specifically, we simulate an LP that adds private capital investments to a pre-specified and realistic public market allocation that is not necessarily the maximum Sharpe ratio allocation (60% to equities and 40% to bonds in our baseline specification). The LP starts with 0% allocations to private capital in 1987 and commits capital to a small set of random private capital funds each vintage year (nine funds in our baseline specification). The annual commitment is done as a fixed fraction of the LP's assets under management, with the LP holding the uncalled committed capital in a liquid asset (public equities in our baseline specification). The proportional annual commitment is calibrated to reach a pre-specified target private capital allocation in steady state (20% in our baseline specification). Nevertheless, the LP actual allocation to public and private markets oscillates over time according to capital calls and distributions (which are the actual capital calls and distributions of the random set of funds the LP commits capital to over time). Given the time series of commitments within a simulation, we obtain the aggregate cash flows and NAVs of the LP each period and calculate the returns to the LP portfolio accordingly (with nowcasted NAVs used to compute private capital returns). Using these returns, we calculate the private capital alpha as the Sharpe ratio increase of the LP portfolio relative to the baseline public market portfolio, normalized appropriately to be in alpha units.

We perform the simulation in the prior paragraph 5,000 times, with the median alpha across simulations provided in the red bars of Figure 1. These numbers reflect the private capital alphas of the three asset classes under analysis. As it is clear from the figure, the private capital alphas are substantially lower than the naive alphas. We also add a bootstrap step to the simulations so that we can obtain confidence intervals that simultaneously account for the uncertainty associated with underdiversification and sampling variation. The 95% alpha confidence intervals are reported on the top of each red bar, with BO being the only asset class that provides a private capital alpha that is positive over the entire confidence interval. These results highlight the importance of accounting for the economic realities of private markets when estimating the alpha of private capital investments.

Our main contribution is to provide a new method to evaluate the performance of private capital investments. In this regard, our innovation is threefold. First, instead of focusing on NPV measures such as the Public Market Equivalent (PME) of Kaplan and Schoar (2005), the Generalized PME (GPME) of Korteweg and Nagel (2016), or a GPME with better fund-level properties (Korteweg and Nagel (2024)), we estimate the alpha of private capital investments. This is important because capital allocation decisions require a comparison of risk-adjusted performance across asset classes. Alpha is the standard risk-adjusted performance metric for most asset classes. As such, having estimates for the alphas of private capital asset classes is essential to guide the allocation decisions of institutional investors. Second, we focus on the risk-adjusted performance of private capital asset classes as opposed to fund-level risk-adjusted performance. While many investment decisions require fund-level risk-adjusted performance metrics, the decision of how much to allocate to private capital asset classes requires risk adjustment at the asset class level. It is not obvious how to translate the cross-fund distribution of NPV measures reported in the literature to the risk-adjusted performance of private capital asset classes. Third, our private capital alpha accounts for the economic realities of private markets. In particular, we account for the underdiversification of investments in private capital as well as the inability of LPs to control their effective allocation to private capital. Moreover, our alpha evaluation considers an allocation to public and private markets that is realistic in the context of typical LPs of private capital funds.

We also contribute to the broader literature evaluating the performance of private capital investments (e.g., Kaplan and Schoar (2005), Pagliari Jr, Scherer, and Monopoli (2005), Harris, Jenkinson, and Kaplan (2014), Brown et al. (2015), Braun, Jenkinson, and Stoff (2017), Korteweg and Sorensen (2017), Brown and Kaplan (2019), Pagliari Jr (2020), Gredil (2022), and Riddiough and Wiley (2022)). Our main contribution to this literature is to provide the first evaluation of private capital as an asset class and from an alpha perspective while accounting for the illiquidity and underdiversification of private capital funds.

The rest of this paper is organized as follows. Section 1 discusses the economics of private capital investing and introduces our private capital alpha. Section 2 details the data and simulations we use to estimate the alphas of the different private capital asset classes. In turn, Section 3 presents our main empirical results and Section 4 explores alternative empirical specifications. Section 5 concludes. The Internet Appendix contains technical derivations.

1 The Private Capital Alpha

In this section, we develop a framework to estimate the alpha of private capital investments. Subsection 1.1 highlights the major reasons why private capital investing is economically different from investing in public markets and Subsection 1.2 explains how we adjust the typical α estimation process to account for the economics of private capital investing.

1.1 The Economics of Private Capital Investing

Investment in private capital is largely done through private capital funds, which are investment vehicles structured as partnerships between the General Partner (GP) and the Limited Partners (LPs). In the typical structure, the investment decisions (such as sourcing, executing, managing, and exiting investments) are delegated by the LPs to the GP. The LPs contractually commit to the private capital fund and can control the size and timing of the commitment. However, they do not control when the committed capital is called by the GP nor the fraction of the committed capital that is eventually called. Once called, the capital is deployed and locked up in the fund until distributed by the GP.

Given the above structure, there are three major characteristics of private capital investing that distinguish it from investing in public markets.

First, while it is relatively easy to invest in a well-diversified portfolio in public markets (e.g., through exchange traded funds), investing in private capital often results in underdiversification (see, e.g., Gredil, Liu, and Sensoy (2024)). Identifying and managing opportunities in private markets requires expertise and a substantial time commitment from the GP. As a consequence, each private capital fund tends to invest in relatively few assets. Moreover, diversifying across funds results in non-trivial monitoring costs for the LPs, which often come in the form of human capital investment. As a consequence, LPs tend to invest in relatively few funds, which themselves invest in relatively few assets, resulting in underdiversification.

Second, private capital investing is illiquid in nature. The contractual commitment between LPs and GP is long (typically in excess of 10 years), with LPs having no control over when capital is called and/or returned while having access to very limited secondary markets.[2] Consequently, allocations to private capital are subject to fluctuations that are partially out of the control of the LPs. For instance, suppose an LP commits 20% of its portfolio to a private capital fund. Until the capital is called, the LP's effective allocation to private capital is 0%. Then, let's say all the capital is called and in one year the private capital fund appreciates by 50% whereas the rest of the LP portfolio has a 0% total return. In this case, the LP's effective allocation to private capital becomes 30%. While this is an extreme example that ignores how diversification limits variability in the effective private capital allocation of LPs, our empirical analysis shows that the portfolio allocation of LPs to private capital oscillates non-trivially around its target allocation even after accounting for realistic diversification.

Third, LPs face mandates and constraints when investing in private capital funds. For instance, investors cannot take short or levered positions in private capital funds as they can do in public markets (e.g., when investing in stocks). Moreover, while in principle the definition of accredited investors is broad enough to allow even (wealthy) retail investors to invest in private capital funds, in practice most private capital investment is done by institutional investors such as pension funds and endowment funds. These institutional investors face internal mandates and more strict constraints than the inability to take short or levered positions in private capital funds. In practice, this could be inferred by an LPs strategic asset allocation as described in their investment policy statement. As such, the overall weight that private capital can take in the typical LP portfolio faces restrictions that are not present for many investors when allocating capital to public markets.

[2]Although the emergence of a secondary market for private capital funds has evolved steadily over the last 20 years, LPs facing liquidity needs who are forced to sell fund stakes during times of market dislocation can typically only do so at a significant discount to NAV and can face reputational penalties with GPs as well. For a detailed analysis of the secondary market for private equity, see Nadauld et al. (2019).

1.2 The Private Capital Alpha Estimation

We now explain the typical α estimation process for liquid assets and how we adjust it to account for the economics of private capital investing described in the previous subsection.

Suppose an institutional investor (which we refer to as LP) has access to a set of public market funds with excess returns given by f_t and is considering to also invest in private capital. To inform the decision process, the LP would like to understand the historical performance of the private capital market relative to the public market funds.

Traditionally (i.e., if private assets were like public assets), the LP would approach this performance evaluation task by checking whether the intercept in a factor regression is zero. Specifically, letting r_t^* represent excess returns on a well-diversified private capital portfolio and $r_{p,t}^* = w_p^{*\prime} f_t$ reflect excess returns on the ex-post maximum Sharpe ratio portfolio that can be formed with f_t, the LP can estimate the alpha intercept in the regression[3]

$$r_t^* \ = \alpha^* \ + \ \beta \cdot r_{p,t}^* \ + \ \epsilon_t \tag{1}$$

and check whether it is zero or not. If $\alpha^* > 0$ ($\alpha^* < 0$), then the historical performance suggests that the LP would have benefited from a positive (negative) allocation to private capital. In contrast, if $\alpha^* = 0$, then there was no benefit to private capital investing to the LP over the period of analysis.

While the alpha estimation framework above is widely applied in academia and in industry, we argue that it doe not properly account for the economics of private capital investing described in Subsection 1.1. To explain how we modify the alpha estimation process to deal with this issue, we start by noting that the α^* in Equation 1 can be alternatively written as

$$\alpha^* \ = sign[w^*] \cdot \sigma[r_t^*] \cdot \sqrt{\left(1 - Cor[r_t^*, r_{p,t}^*]^2\right) \cdot \left| SR[r_{pp,t}^*]^2 \ - \ SR[r_{p,t}^*]^2 \right|} \tag{2}$$

where $SR[x_t] = \mathbb{E}[x_t]/\sigma[x_t]$ is the Sharpe ratio function and $r_{pp,t}^* = w^* \cdot r_t^* + (1 - w^*) \cdot r_{p,t}^*$ is

[3] If there are no arbitrage opportunities when trading on f_t, a Stochastic Discount Factor that perfectly prices f_t can be written as $M_t = a + b \cdot w_p^{*\prime} f_t = a + b \cdot r_{p,t}^*$ (see Chapter 6 in Cochrane (2005)). As such, the intercept from a factor regression that includes all factors in f_t is equivalent to α^* in Equation 1, which uses only the maximum Sharpe ratio portfolio as a factor. We rely on this result as it simplifies our exposition.

the ex-post maximum Sharpe ratio portfolio that can be formed with r_t^* and $r_{p,t}^*$.[4]

Consequently, instead of estimating alpha from the factor regression in Equation 1, we rely on Equation 2 while adjusting it to deal with the underdiversification and illiquidity of private capital investments as well as the mandates and constraints faced by institutional investors. Specifically, we define the private capital alpha as

$$\alpha \ = sign[\Delta] \cdot \sigma[\widetilde{r}_t] \cdot \sqrt{(1 - Cor[\widetilde{r}_t, r_{p,t}]^2) \cdot |SR[r_{pp,t}]^2 \ - \ SR[r_{p,t}]^2|} \tag{3}$$

where $\Delta = SR[r_{pp,t}] - SR[r_{p,t}]$,

$$\widetilde{r}_t \ = (w_{t-1}/w) \cdot r_t + (1 - w_{t-1}/w) \cdot r_{liq,t} \tag{4}$$

and

$$\begin{aligned} r_{pp,t} \ &= w \cdot \widetilde{r}_t \ + \ (1 - w) \cdot r_{p,t} \\ &= w_{t-1} \cdot r_t \ + \ (w - w_{t-1}) \cdot r_{liq,t} \ + \ (1 - w) \cdot r_{p,t} \end{aligned} \tag{5}$$

Equation 3 contains three economic adjustments relative to Equation 2. First, it replaces the ex-post maximum Sharpe ratio portfolio of public assets ($r_{p,t}^*$) with a public market portfolio based on an allocation that is feasible to (and common among) institutional investors ($r_{p,t}$). Second, it replaces the well-diversified private capital portfolio (r_t^*) with an underdiversified private capital portfolio (r_t). And third, it replaces the ex-post maximum Sharpe ratio private capital allocation (w^*) with an effective allocation of w_{t-1}, which targets a feasible allocation of w, but deviates from it given the LP's inability to trade on secondary markets or to control when capital is called or returned. The deviation from the target allocation ($w - w_{t-1}$) is invested in a liquid asset with return $r_{liq,t}$ so that the LP can easily satisfy capital calls and allocate capital distributions that move it away from its target private capital allocation of w. Details on how we measure $r_{p,t}$ and $r_{pp,t}$ in the data are provided in Section

[4] Equation 2, which we prove in Internet Appendix A, is a univariate version of the result in Gibbons, Ross, and Shanken (1989) linking pricing errors to mean-variance efficiency (see also the textbook treatment in Chapter 6.6 of Campbell, Lo, and MacKinlay (1997)). It is used in Gonçalves, Loudis, and Ogden (2024) to study out-of-sample alphas of equity anomaly strategies in public markets.

2.[5]

Another way to think about it is that Equation 2 measures the private capital alpha from the Sharpe ratio increase obtained by moving from the maximum Sharpe ratio allocation in the public market, $r^*_{p,t}$, to the maximum Sharpe ratio allocation that also includes a well-diversified private capital portfolio, $r^*_{pp,t} = w^* \cdot r^*_t + (1 - w^*) \cdot r^*_{p,t}$. However, the inherent underdiversification and illiquidity of private capital markets as well as institutional investors' mandates and constraints likely make these two maximum Sharpe ratio allocations infeasible to many LPs. As such, Equation 3 instead measures the private capital alpha from the Sharpe ratio increase obtained by moving from a feasible public market allocation that is common among LPs, r_p, to another feasible and common allocation among LPs, r_{pp}, but that also includes private capital. Importantly, this allocation that includes private capital is carefully chosen to be consistent with the underdiversification and illiquidity of private capital markets as well as the typical mandates and constraints of institutional investors, $r_{pp,t} = w_{t-1} \cdot r_t + (w - w_{t-1}) \cdot r_{liq,t} + (1 - w) \cdot r_{p,t}$.

2 Data and Measurement for $r_{p,t}$ and $r_{pp,t}$

This section details the data and measurement approach we rely on to construct $r_{p,t}$ and $r_{pp,t}$. Subsection 2.1 explains how we construct our public market portfolio ($r_{p,t}$), Subsection 2.2 details our dataset of private capital funds, and Subsection 2.3 elaborates on our construction of the portfolio with allocation to both public and private markets, $r_{pp,t} = w_{t-1} \cdot r_t + (w - w_{t-1}) \cdot r_{liq,t} + (1 - w) \cdot r_{p,t}$.

[5] Equation 3 also has one technical adjustment relative to Equation 2. Specifically, it replaces $sign(w^*)$ with $sign(\Delta) = sign(SR[r_{pp,t}] - SR[r_{p,t}])$ to make the alpha signs in Equations 2 and 3 consistent. A negative w^* implies a negative α^* in Equation 2 and would also tend to lead to a negative α in Equation 3 through $sign(\Delta)$ because forcing a positive weight on r_t when w^* is negative is detrimental to the final portfolio's Sharpe ratio.

2.1 The Public Market Portfolio: $r_{p,t}$

We construct $r_{p,t}$ from a portfolio that only invests in public equity and fixed-income funds since these are the two major asset classes in public markets. Given our focus on U.S. private capital funds (see next subsection), our $r_{p,t}$ portfolio is based on the U.S. public markets, with allocations to two funds. For the public equity fund, we use the Vanguard Total Stock Market Index Fund (VTSMX) tracking the CRSP U.S. Total Market Index. For the fixed-income fund, we use the Vanguard Total Bond Market Index Fund (VBMFX) tracking the Bloomberg Barclays U.S. Aggregate Bond Index. Monthly return data for these two funds (net-of-fees) are obtained from the Center for Research in Security Prices (CRSP) mutual fund database and cumulated over each quarter to match the frequency of the private capital fund returns.[6]

To obtain excess returns on the equity and bond portfolios, which we label $r_{e,t}$ and $r_{b,t}$, we subtract from their respective net-of-fees returns the risk-free return over the same period. As a proxy for the risk-free return, R_f, we cumulate monthly returns on the 1-month Treasury bill each quarter, with monthly returns obtained from Kenneth French's data library.[7]

In our baseline analysis, the public market portfolio has a 60-40 allocation to equities and bonds (i.e., $r_{p,t} = 0.6 \cdot r_{e,t} + 0.4 \cdot r_{b,t}$). To achieve this allocation, we assume that, at the end of each quarter, the institutional investor rebalances the portfolio to maintain a 60-40 public equity-bond allocation: if $r_{p,t}$ is over(under)allocated to the fixed-income fund, the institutional investor sells (buys) shares of the fixed-income fund and buys (sells) shares of the public equity fund. The choice of 60-40 public equity-bond allocation is consistent with the public market portion of the allocation of typical institutional investors such as U.S. public pension funds. In Subsection 4.1, we explore alternative public equity-bond allocations to understand how the private capital alpha varies across LPs that have different public market

[6] Since VTSMX was first offered to investors in April 1992 (after the beginning of our analysis), we extend it back to 1987 using returns on the CRSP value-weighted index minus a fixed management fee of 30 basis points annually. Although management fees for index mutual funds have declined significantly over the last decades, this level of fees is appropriate for this 1987 to 1992 period.

[7] See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

portfolios.

2.2 The Dataset of Private Capital Funds

Historically, one of the limiting factors on evaluating ex-post performance of private capital funds is the availability of high-quality data on private funds. In this paper, we obtain fund-level data from MSCI Private Capital (previously Burgiss) Universe dataset, which covers a large sample of institutional-quality private funds. MSCI sources its data directly from institutional LPs, and its data has been used in recent studies of fund performance (e.g., Harris, Jenkinson, and Kaplan (2014) and Brown and Kaplan (2019)). For a detailed discussion of the advantages and potential biases of this database, see Brown et al. (2015).

We have access to the complete history of cash flows between LPs and each private capital fund, as well as fund quarterly reported valuations.[8] All data are net-of-fees and carried interest, and so represent the actual LP investment experience. Our sample consists of 5,028 U.S. Buyout (BO), Venture Capital (VC), and Real Estate (RE) private equity funds with vintage years from 1987 to 2022, representing \$3.64 trillion in total fund commitments.[9]

[Table 1 around here]

[Figure 2 around here]

Table 1 provides an overview of our sample by vintage year and fund type. Our sample of 5,028 private capital funds is composed of 1,589 BO funds (with \$2.23 trillion in committed capital), 2,403 VC funds (with \$0.74 trillion in committed capital), and 1,036 RE funds (with \$0.67 trillion in committed capital). There were relatively few funds available in the late 1980s and early 1990s, but investors could gain some within-vintage diversification by

[8] As we detail in Subsection 2.3.6, we also rely on quarterly nowcasted valuations from Brown, Ghysels, and Gredil (2023) to deal with the fact that reported valuations lead to smoothed returns, which in turn lead to understated risk and overstated risk-adjusted performance (see Getmansky, Lo, and Makarov (2004) and Couts, Gonçalves, and Rossi (2024)).

[9] We start in vintage year 1987 because this is the first vintage year with a sufficient number of private capital funds in the MSCI dataset.

the later-half of the 1990s. Overall, private capital funds have experienced substantial growth since the 1980s. Figure 2 provides a quick visualization of this growth in private capital and also of the sample composition across the three fund types. The most important observation is that while most funds in our sample are VC funds, most capital in our sample is committed to BO funds.

2.3 The Public+Private Market Portfolio: $r_{pp,t}$

We consider four versions of $r_{pp,t}$ that vary based on the type of private capital funds the LP allocates capital to: (i) only BO funds, (ii) only VC funds, (iii) only RE funds, or (iv) all three types of funds jointly. For all four $r_{pp,t}$ versions, we assume that the LP has a target allocation of $w = 20\%$ to private capital in its portfolio.[10] Moreover, we assume that capital commitment decisions are made annually even though we evaluate private capital investments based on quarterly returns.

As we discuss in Section 1, the necessary time commitment and monitoring costs involved in private capital investing tend to result in underdiversification. Moreover, the illiquidity of private capital funds makes it impractical to maintain exactly a $w = 20\%$ allocation to private capital because of the uncertainty in future capital calls, distributions, and net asset values of the private capital funds. We accordingly design a strategy that allows for an effective private capital allocation (w_t) that targets $w = 20\%$ to an underdiversified portfolio (r_t), but that deviates from this $w = 20\%$ target allocation at any point in time, with the mismatched weight ($w - w_{t-1}$) allocated to a liquid asset ($r_{liq,t}$).

Specifically, we construct $r_{pp,t} = w_{t-1} \cdot r_t + (w - w_{t-1}) \cdot r_{liq,t} + (1 - w) \cdot r_{p,t}$, with the $r_{liq,t}$, w_t, and r_t components described in the subsections below. Our description focuses on the three versions of $r_{pp,t}$ with a single asset class each (i.e., BO, VC, or RE) and Subsection 2.3.5 explains how we adjust the measurement procedure when constructing the $r_{pp,t}$ version

[10] The choice of $w = 20\%$ is consistent with the average private capital allocation of typical institutional investors such as U.S. public pension funds. In Subsection 4.2, we explore alternative target private capital allocations to help us understand how the private capital alpha varies across LPs that have different private capital allocations.

that includes BO, VC, and RE jointly.

2.3.1 Measuring $r_{liq,t}$

When w_{t-1} deviates from $w = 20\%$, the LP needs to allocate a $w - w_{t-1}$ fraction of its total portfolio to a liquid asset to earn an excess return of $r_{liq,t}$ while having the flexibility to quickly move capital if needed. In our baseline analysis, we use $r_{liq,t} = r_{e,t}$ so that the LP relies on the public equity fund as its liquid asset. The key advantage of using $r_{liq,t} = r_{e,t}$ is that a negative $w - w_{t-1}$ requires no effective negative position on any asset since the large weight on $r_{e,t}$ in the public market portfolio, $r_{p,t}$, ensures that the $r_{pp,t}$ overall weight on $r_{e,t}$ is always positive in our simulations. As such, the LP effectively manages the private capital illiquidity challenge by adjusting the size of its long position on $r_{e,t}$ as needed without ever requiring a short position on any asset.[11]

2.3.2 Measuring w_t, r_t, and $r_{pp,t}$

The LP seeks to reach a steady-state target allocation where 20% of the total portfolio is invested in private capital funds (with the public portion of the portfolio having the same 60-40 split as r_p). To achieve this target $w = 20\%$ allocation, we assume that the LP follows the simple strategy of committing a fixed fraction of its Assets Under Management (AUM), c, each year to private capital funds with inception in that year. For now, we take c as given and explain in Subsection 2.3.4 how we calibrate it.

Given an annual commitment of c, we need to define which funds the LP commits capital to and how much is committed to each fund. We assume that the LP is somewhat (but not completely) diversified in that it allocates capital to 9 funds each year and holds its stakes during the entire lives of the funds, which leads to a typical private capital portfolio with more than 100 funds (e.g., if all funds had the median life of 16 years from Table 1, the private capital portfolio would have 144 funds in steady state).[12] The 9 funds are selected randomly

[11] Subsection 4.6 shows that the private capital alpha is very similar for LPs that instead use $r_{liq,t} = 0$ (i.e., LPs that rely on the risk-free asset for liquidity).

[12] For vintage years with less than 9 funds, we assume that the LP commits capital to all funds available.

from the pool of funds with inception in the respective year (in BO, VC, or RE depending on the $r_{pp,t}$ version).[13] We further assume that the commitment decision for vintage year v is made at the end of the last quarter of calendar year $v-1$ so that the relevant committed capital is set aside (i.e., invested in $r_{liq,t}$) right before year v.[14] Moreover, we assume that the LP value-weights the commitments across funds.[15]

The above paragraph describes the selection of funds and how capital is distributed across funds. These decisions then allow us to calculate the fraction of each fund owned by the LP. Specifically, letting I_v define the set of funds with vintage year v selected by the LP, we have

$$\pi_i^{(v)} = \begin{cases} 0 & if\, i \notin \mathrm{I}_v \\ \pi^{(v)} & if\, i \in \mathrm{I}_v \end{cases} \tag{6}$$

with

$$\pi^{(v)} = \frac{c \cdot \mathrm{AUM}^{(v-1)}}{\mathrm{Size}^{(v)}} \tag{7}$$

where $\pi_i^{(v)}$ is the fraction of fund i owned by the LP (which is the same for all funds in I_v), $\mathrm{Size}^{(v)} = \sum_{j \in \mathrm{I}_v} \mathrm{Size}_j$ reflects the total capital commitment of the funds with vintage year v selected by the LP, and $\mathrm{AUM}^{(v)}$ reflects the LP's AUM at the last quarter of calendar year v (so that the commitment decision associated with vintage year v is made at the end of calendar year $v-1$).

Then, assuming all private capital funds are liquidated no later than 20 years after inception (which is consistent with our data), we calculate w_t and the private capital portfolio

While the focus on 9 funds per vintage year is somewhat arbitrary, Subsection 4.4 provides a comparative statics exercise that varies the number of funds the LP commits capital to annually, which helps us understand how the private capital alpha varies with the level of diversification in the private capital portfolio.

[13] Since our objective is to estimate the overall alpha of private capital investments, we implicitly assume that the LP has access to all available funds but no skill in selecting funds (which justifies the random selection of funds).

[14] Given the typical fundraising timeline for private funds, it is realistic to assume that investors often know which GPs will be raising new funds in the coming year.

[15] Given the large within-vintage heterogeneity in private fund sizes, it is unlikely that an LP can follow an equal-weighted commitment strategy (i.e., commit the same amount to each fund).

quarter t return based on

$$w_t \;=\; \frac{\sum_{v=y_t-20}^{v=y_t} \;\pi^{(v)} \cdot \mathrm{NAV}_t^{(v)}}{\mathrm{AUM}_t} \tag{8}$$

and

$$R_t \;=\; \frac{\sum_{v=y_t-20}^{v=y_t} \;\pi^{(v)} \cdot \left(\mathrm{NAV}_t^{(v)} + \mathrm{CF}_t^{(v)}\right)}{\sum_{v=y_t-20}^{v=y_t} \;\pi^{(v)} \cdot \mathrm{NAV}_{t-1}^{(v)}} \tag{9}$$

where $\mathrm{NAV}_t^{(v)} = \sum_{i\in \mathrm{I}_v} \mathrm{NAV}_{i,t}$, and $\mathrm{CF}_t^{(v)} = \sum_{i\in \mathrm{I}_v} \mathrm{CF}_{i,t}$ reflect the Net Asset Value (NAV) at the end of quarter t and the net cash flow (CF) over quarter t for all funds with vintage year v selected by the LP. Moreover, y_t is the year associated with quarter t.

Finally, we measure excess returns on the LP's private capital portfolio by $r_t = R_t - R_{f,t}$ and $r_{pp,t}$ from Equation 5.

2.3.3 Diversification and Sampling Simulations

Since the choice of the 9 funds that the LP commits capital to each year (i.e., the set I_v) is random, we obtain w_t and r_t (and consequently $r_{pp,t}$) through simulations. Consider the simulation starting at the end of quarter $t-1$, which is the last quarter of year y_{t-1}. We know AUM_{t-1} and w_{t-1} (as well as the history of $\pi^{(v)}$ values for all $v \leq y_{t-1}$). We start by selecting 9 funds of the given category (BO, VC, or RE) randomly to enter $\mathrm{I}_{v=y_t}$ among all funds of the given category with vintage year $v = y_t$ (with equal probability and no replacement) and calculate $\pi^{(v=y_t)}$ based on Equation 7. We then calculate the return on the private capital portfolio based on Equation 9, which allows us to obtain $\mathrm{AUM}_t = (R_{f,t} + r_{pp,t}) \cdot \mathrm{AUM}_{t-1}$ with $r_{pp,t}$ measured from Equation 5. We then have AUM_t, $\mathrm{I}_{v=y_t}$, and $\pi^{(v=y_t)}$, and we calculate w_t based on Equation 8. At this point of the simulation, we know AUM_t and w_t (as well as the history of $\pi^{(v)}$ values for all $v \leq y_t$) and repeat the process above until we reach the last year in the simulation. Importantly, while weights and returns are updated every quarter, we only update $\mathrm{I}_{v=y_t}$, and $\pi^{(v=y_t)}$ at the end of the fourth quarter of each year of the simulation.

We initiate our simulation in t = 1987Q1 with the assumption that $\mathrm{AUM}_{1986Q4} = \$1$ and $w_{1986Q4} = 0$. The assumption that $\mathrm{AUM}_{1986Q4} = \$1$ reflects a normalization while the assumption that $w_{1986\text{Q}4} = 0$ captures the realistic idea that most LPs had little to no private capital allocation in their portfolios in the early 1980s. Finally, our α measurement starts after a “ramp-up” phase of five years (i.e., in 1992Q1) to allow w_t to be non-negligible since $\alpha = 0\%$ would hold by construction if we had $w_t = 0$.[16]

The simulations described above deal with underdiversification. That is, they allow us to study the variability associated purely with the fact that the LP commits capital to a subset of the available funds each year. However, these simulations do not deal with the (more standard) sampling variability linked to the fact that the data we actually observe is a sample of the data that we could have observed. For instance, if the lowest annualized α across the simulations in the prior paragraph is 1% and our sample is from 1987 to 2022, this result implies that an LP that followed the strategy we outline to construct $r_{pp,t}$ over the period from 1987 to 2022 would get an annualized α of at least 1%. However, such a result does not tell us much about the distribution of potential αs that the LP could have gotten if the data from 1987 to 2022 looked differently.

Consequently, we also perform a bootstrap simulation exercise to explore sampling variability. This bootstrap exercise compounds the variability due to underdiversification with the more general sampling variability described in the previous paragraph. Specifically, for each bootstrap simulation, we start by performing the diversification simulation to obtain the time-series for R_t and for

$$\text{NAVG}_t = \frac{\sum_{v=y_t-20}^{v=y_t} \pi^{(v)} \cdot \text{NAV}_t^{(v)}}{\sum_{v=y_t-20}^{v=y_t} \pi^{(v)} \cdot \text{NAV}_{t-1}^{(v)}}, \tag{10}$$

both of which are already random due to underdiversification. We then bootstrap with replacement the dataset composed of ($R_{f,t}$, $r_{liq,t}$, $r_{p,t}$, r_t, NAVG_t) holding fixed the first five years of the dataset (i.e., the ramp-up period).[17] Finally, given the first five years of w_t and

[16] The ramp-up of 5 years is a somewhat arbitrary measurement decision. However, Subsection 4.5 shows that the private capital alpha is similar for alternative reasonable ramp-up periods.

[17] We bootstrap in blocks of 12 quarters so that the sampling variability accounts for potential autocorrelation in returns.

the bootstrapped dataset of ($R_{f,t}$, $r_{liq,t}$, $r_{p,t}$, r_t, NAVG_t), we recursively construct

$$w_t \;=\frac{\sum_{v=y_t-20}^{v=y_t}\;\pi^{(v)}\cdot \text{NAV}_t^{(v)}}{\text{AUM}_t}\;=\;\frac{\text{NAVG}_t\cdot\sum_{v=y_t-20}^{v=y_t}\;\pi^{(v)}\cdot \text{NAV}_{t-1}^{(v)}}{(R_{f,t}+r_{pp,t})\cdot \text{AUM}_{t-1}}\;=\;\frac{\text{NAVG}_t}{(R_{f,t}+r_{pp,t})}\cdot w_{t-1}$$

with $r_{pp,t}$ based on Equation 5.

We refer to the first type of simulations described in this subsection as our "diversification simulations" and the second type as our "sampling simulations". For both of them, we use 5,000 simulations. Consequently, for each statistic reported, we rely on the median value of the respective statistic across simulations. In some parts of our analyses we also report 95% Confidence Intervals (CIs), which are based on the simulation quantiles 2.5% and 97.5%.[18]

2.3.4 Determining c

An annual commitment of c yields a steady state allocation of $w = \lambda \cdot c$ so that we can determine the target commitment, $c = (1/\lambda)\cdot w$, for any given w by estimating the steady state allocation multiplier λ. To calibrate λ, substitute $\pi^{(v)}$ (Equation 7) into w_t (Equation 8) to obtain

$$w_t \;= c\cdot \sum_{v=y_t-20}^{v=y_t} \frac{\text{AUM}^{(v-1)}}{\text{AUM}_t}\cdot\frac{\text{NAV}_t^{(v)}}{\text{Size}^{(v)}} \tag{11}$$

Then, letting g_v represent the annual growth in the LP's AUM (i.e., $\text{AUM}^{(v)} = (1+g_v)\cdot \text{AUM}^{(v-1)}$), Equation 11 (at the last quarter of each year) can be

[18]Since (as explained in Footnote 17), the bootstrap simulations use blocks that represent close to 10% of the bootstrap sample (i.e., each block is 3 years and the 1992 to 2022 period has 31 years), the estimated alphas (and other metrics) are likely biased in each bootstrap sample. To address this issue, we adjust the alpha estimate in each bootstrap sample in a way that the median alpha from the bootstrap simulations matches the median alpha from the diversification simulations (which is not affected by biases related to block simulations). Specifically, letting α_d represent the alpha vector from the diversification simulations and α_b represent the alpha vector from the bootstrap simulations, all statistics reported for the bootstrap simulation rely on the distribution of $\alpha_b - median(\alpha_b) + median(\alpha_d)$. An analogous adjustment is applied to each other performance metric reported from the bootstrap simulations (such as Sharpe ratios and expected returns). Note that our concern is that the bootstrap simulations are biased (not that our estimate for alpha is biased), and thus corrections such as the use of a bias-corrected bootstrap interval would not be appropriate.

written as

$$w_t \;= c \cdot \sum_{v=y_t-20}^{v=y_t} \frac{1}{\prod_{j=v}^{j=y_t}(1+g_j)} \cdot \frac{\text{NAV}_t^{(v)}}{\text{Size}^{(v)}} \tag{12}$$

so that in steady state we have

$$w \;= c \cdot \sum_{h=0}^{h=20} \frac{\text{NAVf}^{(h)}}{(1+g)^{h+1}} \tag{13}$$

where g is the LP's AUM steady state annual growth and $\text{NAVf}^{(h)} = \text{NAV}_t^{(v)}/\text{Size}^{(v)}$ is the steady state NAV as a fraction of fund size, which is a function only of the time since inception, $h = y_t - v$.

Combining Equation 13 with $w = \lambda \cdot c$ implies the steady state allocation multiplier

$$\lambda \;= \sum_{h=0}^{h=20} \frac{\text{NAVf}^{(h)}}{(1+g)^{h+1}} \tag{14}$$

Figure 3(a) plots the total NAV of funds in each asset class relative to their size averaged over time, with the x-axis reflecting the time since the fund's inception year (i.e., the figure displays $\text{NAVf}^{(h)}$ for a strategy in which I_v includes all funds with vintage year v). As the figure shows, only a small fraction of the total commitment is called in the inception year, resulting in a relatively small $\text{NAVf}^{(0)}$ (between 10% and 30%). The total $\text{NAVf}^{(h)}$ grows in the subsequent years through asset returns and capital calls, reaching its peak in $h = 3$ to 5 years depending on the asset class. Then, the $\text{NAVf}^{(h)}$ tends to decline over the subsequent years since distributions outpace capital calls.[19] By year $h = 20$, all funds have been liquidates and thus we have $\text{NAVf}^{(h)} = 0$ for $h \geq 20$.

[Figure 3 around here]

Figure 3(b) displays, for each asset class, λ as a function of g based on Equation 14 by combining the $\text{NAVf}^{(h)}$ estimates in Figure 3(a) with different g values. The allocation multiplier tends to be higher for venture capital funds relative to buyout and real estate

[19]Typically, capital must be called within the first 5 years of a fund's life though exceptions can be made by for certain types of investments such as add-ons. Also, LPs can grant extensions.

funds. In our baseline analysis, we use $g = 8\%$, which implies $\lambda_{BO} = 4.33$, $\lambda_{VC} = 5.97$, and $\lambda_{RE} = 4.19$. Since $c = (1/\lambda) \times 20\%$, we then rely on the annual commitments $c_{BO} = 4.62\%$, $c_{VC} = 3.35\%$, and $c_{RE} = 4.77\%$ for our BO, VC, and RE portfolios, respectively. These commitment values imply the private capital allocation target of $w = 20\%$ regardless of the asset class under analysis (if the underlying AUM growth is 8%).

2.3.5 Adjustments for $r_{pp,t}$ version with BO, VC, and RE

For the $r_{pp,t}$ version in which r_t contains BO, VC, and RE jointly (which we refer to as ALL), we make some modifications in the capital commitment process. Specifically, we assume that the institutional investor still has a target weight of 20% for private capital but distributes this target weights across asset classes in a value-weighted manner (i.e., based on the total capital being raised by each asset class in each vintage year). Specifically, the target weight on asset class k associated with the vintage year v commitment is $w_{k,v} = \phi_{k,v} \cdot 20\%$, where $\phi_{k,v} = \text{Size}_k^{(v)} / (\text{Size}_{\text{BO}}^{(v)} + \text{Size}_{\text{VC}}^{(v)} + \text{Size}_{\text{RE}}^{(v)})$. Consequently, we have the commitment $c_{k,v} = (1/\lambda_k) \times w_{k,v}$ for asset class k in vintage year v. The LP still invests in 9 funds randomly, but requires 3 funds in each asset class. As before, the commitments are value-weighted within each asset class.

2.3.6 Nowcasted NAVs

While the cash flows in our dataset properly reflect proceeds from and to LPs, the NAVs in our dataset (which are reported by the GPs) are only estimates of true NAVs. It is well known that illiquidity and incentive considerations can lead these reported NAVs to be smoothed versions of true NAVs, which in turn leads to understated risk and overstated risk-adjusted performance (see the discussions in Getmansky, Lo, and Makarov (2004) and Couts, Gonçalves, and Rossi (2024)). As such, when constructing the private capital returns in Equation 9 for performance evaluation, we use the nowcasted NAVs from Brown, Ghysels, and Gredil (2023). We use the reported NAVs for all other purposes (e.g., when constructing the weights in Equation 8, when updating AUM_t quarterly, and when calibrating c). The reason is that LPs make their allocation and liquidity decisions based on reported NAVs, not nowcasted NAVs. As such, when calculating private capital returns ($\widetilde{r}_t$) in Equation 4 and portfolios returns (r_{pp}) in Equation 5, our approach effectively assumes the LPs use weights (w_t) that are based on reported NAVs. Similarly, when computing $\text{AUM}_t = \text{AUM}_{t-1} \cdot (r_{pp,t} + R_{f,t})$, we assume the LP uses a $r_{pp,t}$ that is based on reported NAVs.

When reporting our empirical findings, we also provide results that rely entirely on reported NAVs. These results help us identify the effect of NAV smoothing separately from the diversification, illiquidity, and LP constraints considerations we describe in Subsection 1.1.

3 Main Empirical Results

This section details our main empirical results. Subsection 3.1 covers the distribution of private capital fund-level NPVs, Subsection 3.2 presents naive estimates of the alphas of private capital asset classes, and Subsection 3.3 provides our estimates of the alphas of private capital asset classes that account for the economic realities of investing in private capital.

3.1 The NPVs of Private Capital Funds

This subsection explores private capital performance at the fund-level to help us motivate the importance of estimating the private capital α, which focuses on the effect of adding private capital to the typical LP's portfolio.

The usual approach in the literature to measure the performance of a private capital fund is to estimate its Public Market Equivalent (PME) metric introduced in Kaplan and Schoar (2005). Korteweg and Nagel (2016) show that this PME metric can be seen as the Net Present Value (NPV) associated with the fund net cash flows using the Stochastic Discount Factor $\text{SDF}_t = exp(a - b \cdot log(R_{e,t}))$, where $R_{e,t}$ is the gross-return on an equity index and the parameters are restricted to $a = 0$ and $b = 1$.

[Table 2 around here]

Table 2 (Panel A) shows the distribution of these NPVs across our private capital funds in the BO, VC, and RE asset classes.[20] Each fund NPV is normalized by the present value of its negative cash flows (in absolute terms) so that the numbers can be interpreted as the NPVs relative to their respective investment sizes. On average, BO and VC produce positive NPVs (of 18% and 31% respectively) while RE produces a slightly negative NPV (of -6%). Since the performance distribution of private capital funds is very skewed, the median NPVs are perhaps more informative than the average NPVs. From the NPV quantiles, we have that the median NPVs are 13%, -16%, and -7% for BO, VC, and RE, respectively. The contrast between a positive average NPV and a negative median NPV for VC funds is due to the fact that VC performance is extremely skewed, with the 99% quantile of the distribution reflecting a NPV almost 900%.

While the NPVs in Panel A are informative, Korteweg and Nagel (2016) point out that restricting the SDF parameters to $a = 0$ and $b = 1$ can induce a non-trivial bias in the NPVs for funds with market betas that differ from one. They then propose a methodology to estimate a and b by requiring the SDF to perfectly price the equity and risk-free assets while accounting for the typical cash flow patterns in private capital funds. We estimate a and b separately for BO, VC, and RE funds following the methodology in Korteweg and Nagel (2016) and report results in Table 2 (Panel B). The median fund performance in all three asset classes substantially deteriorates relative to Panel A, which suggests that the median fund in each of these asset classes has market beta above one (Korteweg and Nagel (2016) show that the approach in our Panel A overestimates fund NPV when the implicit fund market beta is above one). The average performance of RE funds improves, but this result is driven by the much longer right tail of the RE performance distribution in Panel B relative to Panel A.

[20]For the NPV calculations, we use quarterly net cash flows and quarterly returns. Moreover, we rely only on funds with vintage year of 2012 or earlier so that we have at least ten years of cash flows by year 2022 (the end of our sample period). For funds with positive NAV at the end of 2022, we treat the final NAV as a terminal cash flow.

Since our baseline alpha estimation relies on $r_{p,t} = 0.6 \cdot r_{e,t} + 0.4 \cdot r_{b,t}$ as the public market portfolio, we also estimate the fund-level NPVs using $\text{SDF}_t = exp(a - b \cdot log(R_{p,t}))$, with results reported in Table 2 (Panel C). The overall results suggest that the median performance deteriorates further in all three asset classes relative to Panel B. The average performance of RE funds improves relative to Panel B, but this result is driven by a further increase in the right tail of the RE performance distribution in Panel C relative to Panel B.[21]

Overall, Table 2 yields two clear results. First, most BO, VC, or RE funds provide negative NPV over our sample period once we properly control for public market risk (by estimating a and b instead of setting them to $a = 0$ and $b = 1$). Second, on average, BO and RE funds provide positive NPVs after controlling for public market risk. However, these average NPVs are largely driven by a relatively small fraction of funds with extremely good performance.

These findings provide very little guidance on whether adding private capital to an institutional investor's portfolio is beneficial. On one hand, most funds provide negative NPV. On the other hand, some funds provide very large positive NPVs that may justify investing in their entire asset class. Whether this is in fact the case is unclear, specially considering the underdiversification and illiquidity inherent in private capital investing, which creates non-trivial challenges in terms of achieving the performance that an optimal allocation could potentially deliver.

Moreover, these NPV numbers are not directly comparable to the alpha estimates that LPs obtain for other asset classes. As such, it is hard for LPs to use these NPV numbers to guide decisions related to their target allocation to private equity. Consequently, we argue that providing a way for LPs to estimate the private capital alpha is essential for portfolio allocation decisions.

[21]Note that our bs associated with $log(R_e)$ (in Panel B) are quantitatively in line the bs reported in Korteweg and Nagel (2016, 2024). Our bs associated with $log(R_p)$ (in Panel B) are larger, but this is expected given the lower volatility of $log(R_p)$ relative to $log(R_e)$ as well as the fact that $log(R_p)$ is less correlated with private capital asset classes than $log(R_e)$ is.

3.2 The Naive Private Capital Alpha

This subsection estimates the alpha of BO, VC, and RE markets while treating the underlying funds the same way researchers treat funds that invest in public markets (i.e., without taking into account the issues related to diversification, illiquidity, and LP constraints described in Subsection 1.1). We start by calculating fund returns and aggregating them to asset class returns (i.e., indices) to compute some basic return statistics.[22] We then regress the time series of returns of each asset class index onto public market factors and obtain the respective alpha from the intercept estimate of this regression. We call such an alpha estimate the "naive alpha" as it reflects the alpha that is infeasible to most LPs (obtained from Equations 1 and 2).

[Figure 4 around here]

Before detailing the performance of private capital indices, it is important to note that we obtain index returns using both reported NAVs and nowcasted NAVs. As discussed in Subsection 2.3.6, reported NAVs are often smoothed, which leads to overstated risk-adjusted performance. To demonstrate the smoothing effect empirically, Figure 4 displays return autocorrelations for each index using reported NAVs and nowcasted NAVs. As it is clear from the figure, returns based on reported NAVs display substantial autocorrelations (Panels (a), (c), and (e)) whereas returns based on nowcasted NAVs display much lower autocorrelations (Panels (b), (d), and (f)). While the true return autocorrelations of private capital asset classes are inherently unobservable, it is reasonable to assume that they are not as high as the ones we obtain using reported NAVs. For instance, the 1-quarter return autocorrelations of VC and RE funds is around 0.60 (from reported NAVs), which is substantially higher than

[22]Specifically, we build the quarter t return of each private capital asset class using

$$R_t = \frac{\sum_{v=y_t-20}^{v=y_t} \sum_{i=1}^{N_v} (\text{NAV}_{i,t} + \text{CF}_{i,t})}{\sum_{v=y_t-20}^{v=y_t} \sum_{i=1}^{N_v} \text{NAV}_{i,t-1}}$$

where N_v reflects the number of funds in the given asset class at vintage year v and y_t captures the year associated with quarter t.

what is typically observed in public markets.[23] In contrast, when using nowcasted NAVs, the return autocorrelations of all private capital indices are below 0.40 regardless of asset class and autocorrelation lag.

[Table 3 around here]

Table 3 Panel A provides correlations between excess returns on each public market asset class (bonds and equities) and excess returns on each private market asset class (BO, VC, and RE), using both reported and nowcasted NAVs for private capital. The correlations between r_b and excess returns on private capital asset classes tend to be negative and small, with the largest magnitude being from $Cor(r_b, r_{VC}) = -0.15$. In contrast, r_e has large correlations with private capital asset classes, with correlations of $Cor(r_e, r_{BO}) = 0.75$, $Cor(r_e, r_{VC}) = 0.47$, and $Cor(r_e, r_{RE}) = 0.26$ based on reported NAVs. The correlations with r_e significantly increase once we use nowcasted NAVs, becoming $Cor(r_e, r_{BO}) = 0.91$, $Cor(r_e, r_{VC}) = 0.70$, and $Cor(r_e, r_{RE}) = 0.67$, which indicates that the smoothness of reported NAVs masks the public market risk present in private capital markets. Nevertheless, public markets and private markets are not perfectly correlated, which indicates that BO, VC, and RE can provide non-trivial diversification opportunities relative to public bonds and equities. Since diversification is an important component of the private capital alpha, this result suggests that private capital has the potential to provide a positive alpha.

Table 3 Panel B provides performance statistics for the different asset classes. Focusing on returns from reported NAVs, we see that the annualized Sharpe ratios of BO, VC, and RE are quite large (all substantially larger than the annualized Sharpe ratio of public equities). Similarly, all three private capital asset classes provide positive and large naive alphas relative to public markets whether we use $r_{e,t}$ or $r^*_{p,t} = w^*_e \cdot r_{e,t} + w^*_b \cdot r_{b,t}$ as the public market factor.[24] For instance, the annualized naive alphas relative to $r^*_{p,t}$ are $\alpha_{BO} = 8.3\%$, $\alpha_{VC} = 12.0\%$, and

[23]For instance, we find (in untabulated results) that the return autocorrelations of the equity and bond indices over our sample period is not higher than 0.20 at any autocorrelation lag.

[24]Note that the alpha of private capital relative to $r^*_{p,t} = w^*_e \cdot r_{e,t} + w^*_b \cdot r_{b,t}$ is mathematically identical to the alpha of private capital relative to $r_{e,t}$ and $r_{b,t}$ jointly in a multifactor model (see Footnote 3 for more details).

$\alpha_{RE} = 6.1\%$ (albeit the large VC alpha is statistically insignificant).

As expected, the risk of private capital asset classes increases when we use nowcasted NAVs, which leads to a deterioration in risk-adjusted performance. For instance, annualized volatilities of BO, VC, and RE increase after nowcasting (and so do the correlations with public markets as highlighted above). Consequently, Sharpe ratios and naive alphas decline. Nevertheless, risk-adjusted performance is still quite positive. For instance, all three private capital asset classes continue to have Sharpe ratios above the equity Sharpe ratio. Moreover, naive alphas are still large, with the annualized naive alphas relative to $r^*_{p,t}$ being $\alpha_{BO} = 6.4\%$, $\alpha_{VC} = 7.3\%$, and $\alpha_{RE} = 3.3\%$ (albeit, again, the large VC alpha is statistically insignificant).

3.3 The Private Capital Alpha

As we discussed in Section 1, estimating the alpha of private capital asset classes using traditional alpha analysis (as we do in the prior subsection) does not account for important considerations related to diversification, illiquidity, and LP constraints. In this section, we provide results associated with the private capital alpha from simulations that account for these important considerations (as described in Section 2). As detailed in Subsection 2.3.3, we report median results across 5,000 simulation. Numbers in brackets reflect the 95% confidence interval for the given statistic based on the diversification simulation, which accounts for the uncertainty associated with the fact the LP commits capital to only 9 funds each period (i.e., the LP is underdiversified). Numbers in parentheses provide the 95% confidence interval based on the simulation that compounds the uncertainty associated with underdiversification and sampling.

[Table 4 around here]

Table 4 provides statistics on the performance of each asset class within simulations as well as on a value-weighted investment in all three private capital asset classes (under column "ALL"). The overall performance of private capital asset classes in these simulations is relatively similar to the performance of private capital indexes in Table 3. Nevertheless, the next to last row of each panel highlights the effect of underdiversification, which is not present in private capital indices. Underdiversification creates uncertainty even if we ignore sampling variability, with different hypothetical LPs having different performance simply due to the different funds they commit capital to each year (whereas with indices we implicitly consider an investment all funds available each period). For instance, while the typical LP obtains a Sharpe ratio of 0.65 over the sample period from its value-weighted private capital investments, about 5% of the LPs experience Sharpe ratios below 0.54 or above 0.76 for the same strategy.

[Figure 5 around here]

Illiquidity affects the LPs ability to control their portfolio allocations to private capital (i.e., what weight of their portfolios is invested in private capital). Figure 5 shows the time-series of private capital weights (w_t) in the simulations. The black line provides the median w_t while the blue and red lines provide the w_t at quantiles 1% and 99% of the w_t distribution each period. In all cases, the contribution strategy targets 20% w_t at steady state but the actual w_t oscillates over time. The first few years reflect the ramp-up period, with the private capital allocation starting at 0% and increasing towards 20%. Later periods reflect large oscillations due to the inability of the LP to control GP's capital calls and distributions.

Table 5 Panel A shows the cross-simulation median for the average w_t after the ramp-up period of each simulation. The average w_t is relatively close to the 20% target for BO (16.1%), VC (21.3%), and ALL (16.6%). However, the average w_t is far from the 20% target allocation in the case of RE (12.5%). Deviations from target are due to two reasons. First, LPs are not at steady state allocation after a ramp-up period of 5 years, and thus it is natural for the average w_t over our sample to be below the 20% target allocation. Second, the average AUM growth in the simulations (i.e., $R_f + \mathbb{E}[r_j]$) is below the assumed 8% for RE and above the assumed 8% for VC (with BO and ALL being close to the 8% assumption). As such, the steady state w_t for RE is lower than 20% while the steady state w_t for VC is higher than 20%. In Subsection 4.5 we study alternative ramp-up periods while in Subsection 4.3 we explore alternative AUM growth assumptions for the commitment calibration.

[Table 5 around here]

Table 5 Panels B and C provide the performance of each portfolio that combines public and private investments (r_{pp}), with the corresponding private capital alpha obtained from Equation 3 applied within each simulation. From Panel B, we have that all three private capital asset classes yield positive and annual alphas in most simulations if returns are obtained from reported NAVs. In particular, the 95% confidence intervals of all alphas in Panel B contains only positive numbers (except for the VC confidence interval, but even then it mostly covers positive values). In contrast, Panel C shows that, with returns based on nowcasted NAVs, only BO provides positive annual alphas in all simulations within the confidence interval (with the median alpha being 2.1% and the 95% confidence interval ranging from 0.0% to 4.0%). VC provides a large median annual alpha (of 3.0%), but the 95% confidence interval goes from -5.4% to 13.6%, leading to no statistically reliable conclusion on the private capital alpha of VC over our sample period. RE provides a slightly negative alpha (of -0.7%), but the 95% confidence interval goes from -1.5% to 1.7%. Almost mimicking the BO behavior, the value-weighted strategy that combines the three private capital asset classes provides a positive annual alpha in the vast majority of the simulations (the median alpha is 2.0% and the 95% confidence interval ranges from -0.5% to 6.8%).

[Figure 6 around here]

The alpha distributions across simulations can be visualized in Figure 6. As it is clear from the figure, the BO and ALL alpha distributions mostly cover positive values. In contrast, the VC distribution covers a lot of negative values even though it is centered at a higher alpha than the BO distribution. The RE distribution is centered at a slightly negative alpha, but it covers a large range of positive alphas as well.

Overall, the results suggest that the private capital alpha is substantially lower than the naive alpha analysis of the prior subsection indicates. In particular, over our sample period, only BO provided statistically reliable alpha (albeit much lower than the respective naive alpha).

4 Results based on Alternative Empirical Decisions

In this section, we report results on the private capital alpha of the three asset classes we study under alternative empirical decisions. These result serve as comparative statics analyses. In all cases, we report only private capital alphas from returns based on nowcasted NAVs.

4.1 Alternative Public Market Portfolios

[Figure 7 around here]

Our baseline analysis relies on $r_{p,t} = w_e \cdot r_{e,t} + (1 - w_e) \cdot r_{b,t}$ with $w_e = 0.6$. Figure 7 provides the private capital alpha cross-simulation distribution under $w_e = 0.4$ and $w_e = 0.8$. The results under $w_e = 0.8$ are very similar to the baseline results. In contrast, the VC and RE alphas display a non-trivial decline if $w_e = 0.4$. However, the BO alpha increases slightly, keeping the ALL alpha quite stable across the different w_e values we explore.

4.2 Alternative Target Allocations to Private Capital

[Figure 8 around here]

Our baseline analysis relies on the private capital target weight of $w = 0.2$. Figure 8 provides the private capital alpha cross-simulation distribution under $w = 0.1$ and $w = 0.3$. The results are very similar to our baseline results. In particular, BO provided a statistically reliable positive alpha over the sample period, while VC and RE did not. Moreover, only a small fraction of simulations (but higher than 5%) led to a negative alpha for the value-weighted strategy that combines all three private capital asset classes.

4.3 Alternative AUM Growth for Commitment Calibration

[Figure 9 around here]

Our baseline analysis calibrates commitments based on an assumed AUM growth of $g = 8\%$. Figure 9 provides the private capital alpha cross-simulation distribution under $g = 6\%$ and $g = 10\%$. The results are very similar to our baseline results. In particular, BO provided a statistically reliable positive alpha over the sample period, while VC and RE did not. Moreover, only a small fraction of simulations (but higher than 5%) led to a negative alpha for the value-weighted strategy that combines all three private capital asset classes.

4.4 Alternative Levels of Private Capital Diversification

[Figure 10 around here]

Our baseline analysis calibrates commitments based on LPs that commit capital to 9 funds each year. Figure 10 provides the private capital alpha cross-simulation distribution under alternative strategies in which the LP commits to either 3 funds each year or to all funds with vintage in that year. The main effect of the number of funds is that alpha uncertainty declines as the strategy becomes more diversified. However, the overall results are very similar to our baseline results. In particular, BO provided a statistically reliable positive alpha over the sample period, while VC and RE did not. Moreover, only a small fraction of simulations (but higher than 5%) led to a negative alpha for the value-weighted strategy that combines all three private capital asset classes.

4.5 Alternative Ramp-up Periods

[Figure 11 around here]

Our baseline analysis estimates alphas after a ramp-up period of 5 years. Figure 11 provides the private capital alpha cross-simulation distribution under ramp-up periods of 1 year and 10 years. Using a ramp-up period of one year improves the private capital alpha of all three asset classes while using a ramp-up period of 10 years deteriorates the private capital alpha of all three asset classes (albeit the BO private capital alpha remains positive in the vast majority of the simulations). The reason is that private capital funds with vintage year in the late 1980s and most of the 1990s performed better than the private capital funds with later vintage years. Hence, including early funds in the alpha estimation improves the private capital alpha.

4.6 Alternative Liquid Asset

[Figure 12 around here]

Our baseline analysis estimates alphas using r_e as the liquid asset LPs invest their committed capital not yet called. Figure 11 provides the private capital alpha cross-simulation distribution under an alternative approach in which the LP invests in Treasury bills as their liquid asset. The results are relatively similar to our baseline results.

5 Conclusion

We combine a large sample of 5,028 U.S. buyout, venture capital, and real estate funds from 1987 to 2022 to estimate the alphas of private capital asset classes under realistic simulations that account for the illiquidity and underdiversification in private markets as well as the portfolio allocation of typical limited partners. We find that buyout as an asset class has provided a positive and statistically significant alpha during our sample period. In contrast, over our sample period, the venture capital alpha was positive but statistically unreliable and the real estate alpha was, if anything, negative.

Our work provides a novel way to quantify the risk-adjusted performance of private capital investments. Our new method focuses on alphas (as opposed to NPV metrics), applies to private capital asset classes (as opposed to single funds), and accounts for the economic realities of investing in private markets. These three features make our private capital alpha an attractive risk-adjusted performance metric.

Moreover, our simulation method is flexible enough so that it can be easily adjusted to study questions outside the scope of this paper. For instance, our simulations can be adjusted to study the effect of liquidity shocks on the attractiveness of private capital investments.

We hope this and other important questions can be addressed in future research building on our private capital alpha simulations.

References

Begenau, J., P. Liang, and E. Siriwardane (2023). “The Rise in Alternatives”. Working Paper.

Braun, R., T. Jenkinson, and I. Stoff (2017). “How persistent is private equity performance? Evidence from deal-level data”. In: *Journal of Financial Economics* 123.2, pp. 273–291.

Brown, G. W., E. Ghysels, and O. Gredil (2023). “Nowcasting Net Asset Values: The Case of Private Equity”. In: *Review of Financial Studies* 36.3, pp. 945–986.

Brown, G. W., O. R. Gredil, and S. N. Kaplan (2019). “Do private equity funds manipulate reported returns?” In: *Journal of Financial Economics* 132, pp. 267–297.

Brown, G. W., R. S. Harris, T. Jenkinson, S. N. Kaplan, and D. T. Robinson (2015). “What Do Different Commercial Data Sets Tell Us About Private Equity Performance?”

Brown, G. W. and S. N. Kaplan (2019). “Have Private Equity Returns Really Declined?” In: *Journal of Private Equity* 22, pp. 11–18.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton University Press.

Cochrane, J. H. (2005). *Asset Pricing*. Revised Edition. Princeton University Press.

Couts, S., A. S. Gonçalves, and A. Rossi (2024). “Unsmoothing Returns of Illiquid Funds”. In: *Review of Financial Studies* Forthcoming.

Getmansky, M., A. W. Lo, and I. Makarov (2004). “An econometric model of serial correlation and illiquidity in hedge fund returns”. In: *Journal of Financial Economics* 74.3, pp. 529–609.

Gibbons, M. R., S. A. Ross, and J. Shanken (1989). “A Test of the Efficiency of a Given Portfolio”. In: *Econometrica* 57.5, pp. 1121–1152.

Gonçalves, A. S., J. A. Loudis, and R. E. Ogden (2024). “Out-of-Sample Alphas Post Publication”. Working Paper.

Gredil, O., Y. Liu, and B. A. Sensoy (2024). “Diversifying Private Equity”. Working Paper.

Gredil, O. R. (2022). “Do Private Equity Managers Have Superior Information on Public Markets?” In: *Journal of Financial and Quantitative Analysis* 57.1, pp. 321–358.

Harris, R. S., T. Jenkinson, and S. N. Kaplan (2014). “Private Equity Performance: What Do We Know?” In: *Journal of Finance* 69.5, pp. 1851–1882.

Kaplan, S. N. and A. Schoar (2005). “Private Equity Performance: Returns, Persistence, and Capital Flows”. In: *Journal of Finance* 60.4, pp. 1791–1823.

Korteweg, A. and S. Nagel (2016). “Risk-Adjusting the Returns to Venture Capital”. In: *Journal of Finance* 71.3, pp. 1437–1470.

Korteweg, A. and S. Nagel (2024). “Risk-Adjusted Returns of Private Equity Funds: A New Approach”. In: *Review of Financial Studies* Forthcoming.

Korteweg, A. and M. Sorensen (2017). “Skill and luck in private equity performance”. In: *Journal of Financial Economics* 124.3, pp. 535–562.

Nadauld, T. D., B. A. Sensoy, K. Vorkink, and M. S. Weisbach (2019). “The liquidity cost of private equity investments: Evidence from secondary market transactions”. In: *Journal of Financial Economics* 132, pp. 158–181.

Pagliari Jr, J. L. (2020). “Real Estate Returns by Strategy: Have Value-Added and Opportunistic Funds Pulled Their Weight?” In: *Real Estate Economics* 48.1, pp. 89–134.

Pagliari Jr, J. L., K. A. Scherer, and R. T. Monopoli (2005). “Public versus private real estate equities: a more refined, long-term comparison”. In: *Real Estate Economics* 33.1, pp. 147–187.

Riddiough, T. J. and J. A. Wiley (2022). “Private Funds for Ordinary People: Fees, Flows, and Performance”. In: *Journal of Financial and Quantitative Analysis* 57.8, pp. 3252–3280.

Table 1

The MSCI Private Capital Funds used in our Analysis

This table provides the number of funds and total commitment to funds (in $ millions) by vintage year and private capital asset class. The table also provides the median life of funds in each vintage year. We use the MSCI data while keeping only funds with a US geographical focus. If there are less than five funds in a given asset class and vintage year, we do disclose the respective total commitment and median life of funds because of confidentiality restrictions with MSCI. Subsection 2.2 provides data details and a description of the results in this table.

Vintage	Buyout			Venture Capital			Real Estate			All		
Year	$	N	T	$	N	T	$	N	T	$	N	T
1987	8,371	8	17	1,567	29	13	***	4	***	11,048	41	15
1988	6,329	11	14	2,490	30	15	1,667	6	13	10,485	47	15
1989	3,129	11	17	3,389	29	16	904	8	15	7,422	48	16
1990	2,306	8	17	838	14	15	***	3	***	3,599	25	15
1991	***	3	***	522	7	15	***	2	***	2,133	12	15
1992	4,260	10	14	1,590	18	15	***	4	***	6,926	32	15
1993	5,437	12	17	2,045	23	15	593	5	14	8,075	40	15
1994	7,132	19	16	1,784	21	16	2,726	8	15	11,642	48	16
1995	17,829	28	15	3,101	29	16	2,820	10	15	23,749	67	15
1996	5,310	18	13	2,654	22	17	2,419	8	16	10,383	48	16
1997	29,673	31	16	6,573	51	15	6,040	17	14	42,286	99	15
1998	41,756	44	16	11,077	59	17	12,240	27	14	65,072	130	16
1999	29,388	35	17	30,089	97	17	5,342	14	14	64,819	146	17
2000	62,546	54	17	43,559	127	19	6,415	14	15	112,520	195	18
2001	22,226	31	17	22,277	62	19	4,420	16	13	48,923	109	18
2002	15,234	21	17	5,615	23	17	5,900	16	14	26,749	60	16
2003	22,323	25	16	5,834	25	18	7,154	15	14	35,311	65	17
2004	35,276	44	16	10,996	45	18	9,192	29	13	55,464	118	17
2005	50,038	57	17	18,094	66	17	21,132	44	15	89,264	167	17
2006	147,096	67	16	30,306	89	16	28,156	41	13	205,558	197	16
2007	114,882	69	15	24,070	79	15	59,535	66	14	198,487	214	15
2008	95,920	68	15	19,354	65	14	15,657	32	13	130,930	165	14
2009	21,916	23	14	11,932	29	14	8,926	16	13	42,774	68	14
2010	22,814	29	12	10,678	34	12	11,035	20	12	44,527	83	12
2011	67,145	52	-	14,536	51	-	32,427	33	-	114,108	136	-
2012	47,238	49	-	18,108	61	-	15,352	33	-	80,698	143	-
2013	71,263	47	-	13,328	58	-	29,401	46	-	113,992	151	-
2014	84,055	74	-	28,711	98	-	31,915	49	-	144,681	221	-
2015	71,188	53	-	26,742	105	-	48,307	51	-	146,237	209	-
2016	121,754	84	-	21,692	88	-	34,009	47	-	177,454	219	-
2017	102,640	67	-	35,533	117	-	33,004	46	-	171,176	230	-
2018	152,166	88	-	41,729	123	-	25,835	54	-	219,730	265	-
2019	214,115	103	-	42,949	143	-	61,524	69	-	318,588	315	-
2020	153,243	79	-	63,557	159	-	36,143	49	-	252,942	287	-
2021	240,890	99	-	98,665	186	-	44,381	65	-	383,936	350	-
2022	134,857	68	-	63,897	141	-	62,393	69	-	261,148	278	-
Average	62,022	44	16	20,552	67	16	18,615	29	14	101,190	140	16
Total	2,232,803	1,589	-	739,879	2,403	-	670,157	1,036	-	3,642,838	5,028	-

Table 2

NPVs of Private Capital Funds

This table shows the cross-fund distribution of Net Present Values (NPVs) using the Generalized Public Market Equivalent (GPME) metric of Korteweg and Nagel (2016), which is based on an SDF of the form $SDF_t = exp(a - b \cdot f_t)$, where f_t is the relevant risk factor. Panel A provides the PME metric of Kaplan and Schoar (2005) (in NPV terms), which is equivalent to the GPME with $a = 0$, $b = 1$, and $f_t = log(R_{e,t})$. Panel B provides the the GPME metric with a and b estimated separately for each asset class (without constraints), and $f_t = log(R_{e,t})$. Panel C provides the GPME metric with a and b estimated separately for each asset class (without constraints), and $f_t = log(R_{p,t})$ with $R_{p,t} = 0.6 \cdot R_{e,t} + 0.4 \cdot R_{b,t}$. All NPV values are normalized by the present value of the fund negative cash flows (in absolute terms) so that the numbers can be interpreted as the NPVs relative to their respective investment sizes. Moreover, the table covers all three private capital asset classes we study: buyout (BO), venture capital (VC), real estate (RE). Subsection 2.2 provides data details and Subsection 3.1 provides a description of the results in this table.

PANEL A: $\mathrm{SDF}_t = exp(0 - 1 \cdot log(R_{e,t}))$

	a	b	N	Mean	Q1%	Q5%	Q10%	Q25%	Q50%	Q75%	Q90%	Q95%	Q99%
BO	0.00	1.00	827	0.18	-0.83	-0.55	-0.36	-0.13	0.13	0.43	0.80	1.05	1.53
VC	0.00	1.00	1,185	0.31	-0.94	-0.83	-0.72	-0.49	-0.16	0.37	1.51	2.85	8.79
RE	0.00	1.00	491	-0.06	-0.95	-0.77	-0.61	-0.33	-0.07	0.20	0.45	0.69	1.05

PANEL B: $\mathrm{SDF}_t = exp(a - b \cdot log(R_{e,t}))$

	a	b	N	Mean	Q1%	Q5%	Q10%	Q25%	Q50%	Q75%	Q90%	Q95%	Q99%
BO	0.05	3.79	827	0.14	-0.88	-0.72	-0.59	-0.40	-0.12	0.42	1.16	1.63	3.79
VC	0.04	3.50	1,185	-0.06	-0.97	-0.89	-0.83	-0.67	-0.38	0.09	0.81	1.66	5.15
RE	0.06	4.10	491	0.14	-0.97	-0.87	-0.79	-0.59	-0.35	0.20	1.80	2.96	7.77

PANEL C: $\mathrm{SDF}_t = exp(a - b \cdot log(R_{p,t}))$

	a	b	N	Mean	Q1%	Q5%	Q10%	Q25%	Q50%	Q75%	Q90%	Q95%	Q99%
BO	0.12	8.67	827	0.06	-0.93	-0.84	-0.71	-0.52	-0.23	0.33	1.09	1.76	4.63
VC	0.11	8.74	1,185	-0.24	-0.98	-0.95	-0.91	-0.80	-0.56	-0.11	0.69	1.42	3.80
RE	0.14	8.97	491	0.20	-0.98	-0.91	-0.86	-0.67	-0.32	0.18	1.91	3.38	9.46

Table 3
Summary Statistics for Different Asset Classes: Correlations and Performance

This table provides basic risk and reward statistics for value-weighted public and private market indices. The public market indices cover equities (e) and bonds (b). The private market indices cover buyout (BO), venture capital (VC), real estate (RE). The returns on the private market indices are obtained using reported NAVs (left side of the table) and nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing (right side of the table). Panel A provides return correlations between the different asset classes. Panel B provides basic performance metrics. $\mathbb{E}[r]$ is the average excess return, $\sigma[r]$ is the excess return volatility, $SR = \mathbb{E}[r]/\sigma[r]$ is the Sharpe ratio, $\alpha^{(e)}$ is the alpha relative to r_e, and $\alpha^{(e,b)}$ is the alpha relative to r_e and r_b (which is equivalent to the alpha relative to the maximum Sharpe ratio portfolio of r_e and r_b). We refer to the alphas in this table as "naive alphas" since they do not account for the economic realities of private capital investing (beyond NAV smoothing). Subsection 2.2 provides data details and Subsection 3.2 provides a description of the results in this table.

PANEL A: Return Correlations Between Asset Classes

	Returns using Reported NAVs					Returns using Nowcasted NAVs				
	r_e	r_b	r_{BO}	r_{VC}	r_{RE}	r_e	r_b	r_{BO}	r_{VC}	r_{RE}
r_e	1.00					1.00				
r_b	-0.06	1.00				-0.06	1.00			
r_{BO}	0.75	-0.10	1.00			0.91	-0.14	1.00		
r_{VC}	0.47	-0.15	0.56	1.00		0.70	-0.13	0.66	1.00	
r_{RE}	0.26	-0.15	0.49	0.22	1.00	0.67	-0.13	0.68	0.47	1.00

PANEL B: Performance of Each Asset Class

	Returns using Reported NAVs					Returns using Nowcasted NAVs				
	r_e	r_b	r_{BO}	r_{VC}	r_{RE}	r_e	r_b	r_{BO}	r_{VC}	r_{RE}
$\mathbb{E}[r_j]$	8.3%	2.2%	11.9%	15.5%	6.5%	8.3%	2.2%	12.0%	14.4%	5.8%
$\sigma[r_j]$	16.8%	4.0%	10.8%	22.0%	8.6%	16.8%	4.0%	14.0%	24.5%	9.2%
SR_j	0.49	0.55	1.10	0.71	0.76	0.49	0.55	0.86	0.59	0.64
$\alpha_j^{(e)}$	0.0%	2.3%	8.0%	10.5%	5.4%	0.0%	2.3%	5.7%	5.9%	2.8%
	(0.00)	(2.16)	(4.25)	(1.45)	(2.01)	(0.00)	(2.16)	(4.08)	(0.91)	(1.56)
$\alpha_j^{(e,b)}$	0.0%	0.0%	8.3%	12.0%	6.1%	0.0%	0.0%	6.4%	7.3%	3.3%
	(0.00)	(0.00)	(4.48)	(1.35)	(2.57)	(0.00)	(0.00)	(4.40)	(1.00)	(1.97)

Table 4

The Different Asset Classes in Simulations

This table provides basic risk and reward statistics for value-weighted public market indices as well as private capital investments that are realistic from the perspective of the typical Limited Partner (LP). The public market indices cover equities (e) and bonds (b). The private market investments cover buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details on these simulations). For each statistic, we report median results across 5,000 simulation. Numbers in brackets reflect the 95% confidence interval for the given statistic based on the diversification simulation, which accounts for the uncertainty associated with the fact that the LP commits capital to only 9 funds each period. Numbers in parentheses provide the 95% confidence interval based on the simulations that compound the uncertainty associated with underdiversification and sampling variation. $\mathbb{E}[r]$ is the average excess return, $\sigma[r]$ is the excess return volatility, and $SR = \mathbb{E}[r]/\sigma[r]$ is the Sharpe ratio. Subsection 2.2 provides data details and Subsection 3.3 provides a description of the results in this table.

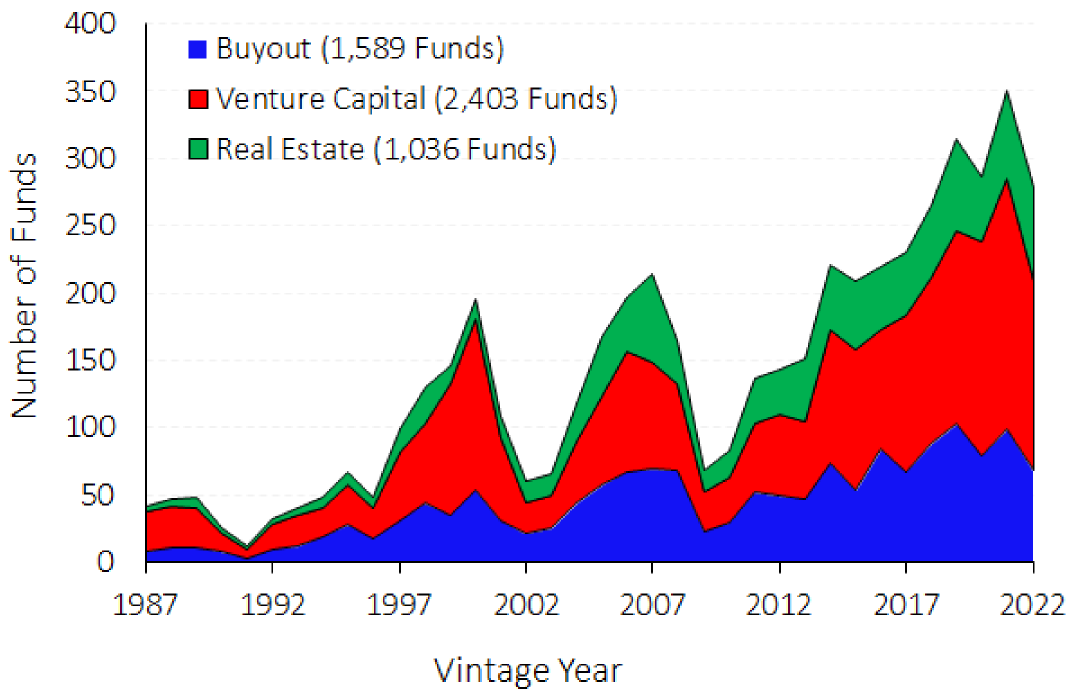
	Equity	**Bond**	**Private (r)**			
	(r_e)	**(r_b)**	**BO**	**VC**	**RE**	**ALL**
PANEL A: Returns using Reported NAVs						
$\mathbb{C}or[r_j, r_e]$	1.00	-0.06	0.70	0.46	0.27	0.58
$\mathbb{C}or[r_j, r_b]$	-0.06	1.00	-0.14	-0.16	-0.16	-0.16
Average Return ($\mathbb{E}[r_j]$)	8.3%	2.2%	13.2%	16.4%	7.0%	13.1%
Volatility ($\sigma[r_j]$)	16.8%	4.0%	10.8%	23.4%	8.3%	12.1%
$SR_j = \mathbb{E}[r_j]/\sigma[r_j]$	0.49	0.55	1.22	0.70	0.84	1.08
	[0.49 ; 0.49]	[0.55 ; 0.55]	[1.07 ; 1.39]	[0.62 ; 0.79]	[0.73 ; 0.96]	[0.87 ; 1.29]
	(0.09 ; 0.91)	(0.19 ; 0.93)	(0.59 ; 2.02)	(0.10 ; 1.46)	(0.03 ; 2.19)	(0.47 ; 1.92)
PANEL B: Returns using Nowcasted NAVs						
$\mathbb{C}or[r_j, r_e]$	1.00	-0.06	0.87	0.67	0.66	0.77
$\mathbb{C}or[r_j, r_b]$	-0.06	1.00	-0.14	-0.13	-0.09	-0.12
Average Return ($\mathbb{E}[r_j]$)	8.3%	2.2%	11.6%	14.3%	4.6%	10.6%
Volatility ($\sigma[r_j]$)	16.8%	4.0%	14.4%	25.0%	9.6%	16.3%
$SR_j = \mathbb{E}[r_j]/\sigma[r_j]$	0.49	0.55	0.81	0.57	0.48	0.65
	[0.49 ; 0.49]	[0.55 ; 0.55]	[0.72 ; 0.90]	[0.52 ; 0.63]	[0.40 ; 0.56]	[0.54 ; 0.76]
	(0.09 ; 0.91)	(0.19 ; 0.93)	(0.36 ; 1.33)	(0.02 ; 1.25)	(-0.10 ; 1.12)	(0.17 ; 1.23)

Table 5
The Total Portfolio (Public + Private) in Simulations

This table provides basic risk and reward statistics for a baseline public market portfolio and a portfolio that combines this public market portfolio with private capital investments. The public market portfolio is composed of 60% equities (e) and 40% bonds (b). The private market investments cover buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details on these simulations). For each statistic, we report median results across 5,000 simulation. Numbers in brackets reflect the 95% confidence interval for the given statistic based on the diversification simulation, which accounts for the uncertainty associated with the fact that the LP commits capital to only 9 funds each period. Numbers in parentheses provide the 95% confidence interval based on the simulations that compound the uncertainty associated with underdiversification and sampling variation. $\mathbb{E}[r]$ is the average excess return, $\sigma[r]$ is the excess return volatility, and $SR = \mathbb{E}[r]/\sigma[r]$ is the Sharpe ratio. The private capital alpha is obtained from Equation 3 in each simulation. Subsection 2.2 provides data details and Subsection 3.3 provides a description of the results in this table.

	Public	**Public + Private (r_{pp})**			
	(r_p)	**BO**	**VC**	**RE**	**ALL**
PANEL A: Portfolio Allocation					
Public Equity	60.0%	48.0%	48.0%	48.0%	48.0%
Fixed Income	40.0%	32.0%	32.0%	32.0%	32.0%
Private Capital (w_t)	0.0%	16.1%	21.3%	12.5%	16.6%
PANEL B: Portfolio Performance (Returns using Reported NAVs)					
Average Return ($\mathbb{E}[r_j]$)	5.9%	7.1%	8.6%	6.2%	7.1%
Volatility ($\sigma[r_j]$)	10.1%	9.9%	11.3%	9.7%	9.9%
$SR_j = \mathbb{E}[r_j]/\sigma[r_j]$	0.58	0.72	0.76	0.64	0.72
	-	0.14	0.18	0.06	0.14
$\Delta SR = SR_{pp} - SR_p$	-	[0.12 ; 0.16]	[0.14 ; 0.21]	[0.05 ; 0.07]	[0.11 ; 0.17]
	-	(0.08 ; 0.25)	(0.03 ; 0.50)	(0.02 ; 0.14)	(0.07 ; 0.28)
	-	3.4%	10.3%	2.1%	4.3%
Private Capital Alpha (α)	-	[2.8% ; 4.0%]	[8.3% ; 12.3%]	[1.9% ; 2.4%]	[3.2% ; 5.9%]
	-	(1.3% ; 5.5%)	(-0.4% ; 22.8%)	(0.0% ; 4.2%)	(0.8% ; 8.1%)
PANEL C: Portfolio Performance (Returns using Nowcasted NAVs)					
Average Return ($\mathbb{E}[r_j]$)	5.9%	6.9%	7.4%	5.8%	6.6%
Volatility ($\sigma[r_j]$)	10.1%	10.7%	12.3%	10.2%	10.8%
$SR_j = \mathbb{E}[r_j]/\sigma[r_j]$	0.58	0.64	0.60	0.57	0.61
	-	0.06	0.02	-0.01	0.03
$\Delta SR = SR_{pp} - SR_p$	-	[0.04 ; 0.08]	[-0.01 ; 0.05]	[-0.02 ; 0.00]	[0.01 ; 0.05]
	-	(0.02 ; 0.13)	(-0.06 ; 0.32)	(-0.03 ; 0.04)	(-0.01 ; 0.13)
	-	2.1%	3.0%	-0.7%	2.0%
Private Capital Alpha (α)	-	[1.6% ; 2.6%]	[-1.6% ; 4.6%]	[-1.0% ; 0.3%]	[1.0% ; 3.0%]
	-	(0.0% ; 4.0%)	(-5.4% ; 13.6%)	(-1.5% ; 1.7%)	(-0.5% ; 6.8%)

(a) Number of Funds by Fund Type



(b) Committed Capital by Fund Type

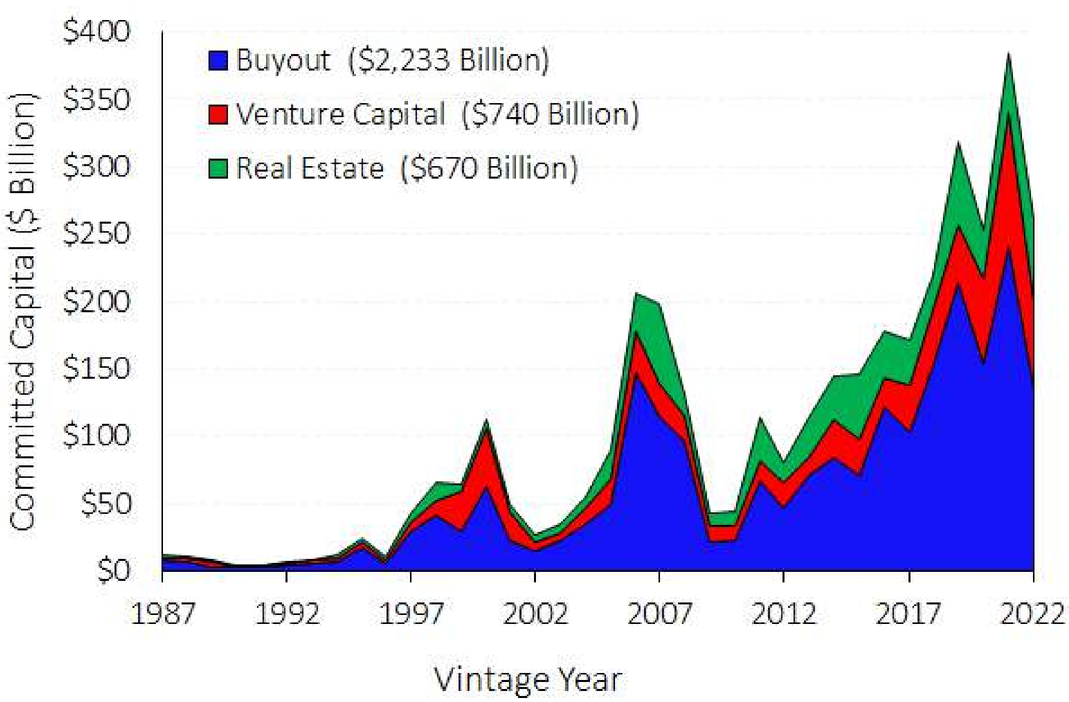
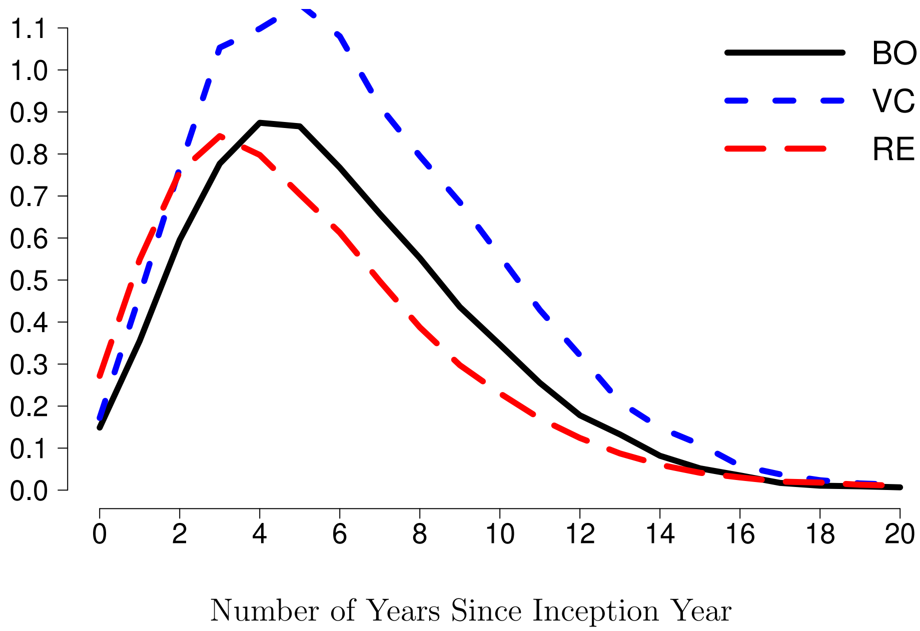


Figure 2
Number of Funds and Committed Capital by Fund Type

This figure plots the number of funds (Panel (a)) and the total commitment to funds in $ billions (Panel (b)) by vintage year and private capital asset class. We use the MSCI data while keeping only funds with a US geographical focus. Subsection 2.2 provides data details and a description of the results in this figure.

(a) NAV / Size



(b) Allocation Multiplier

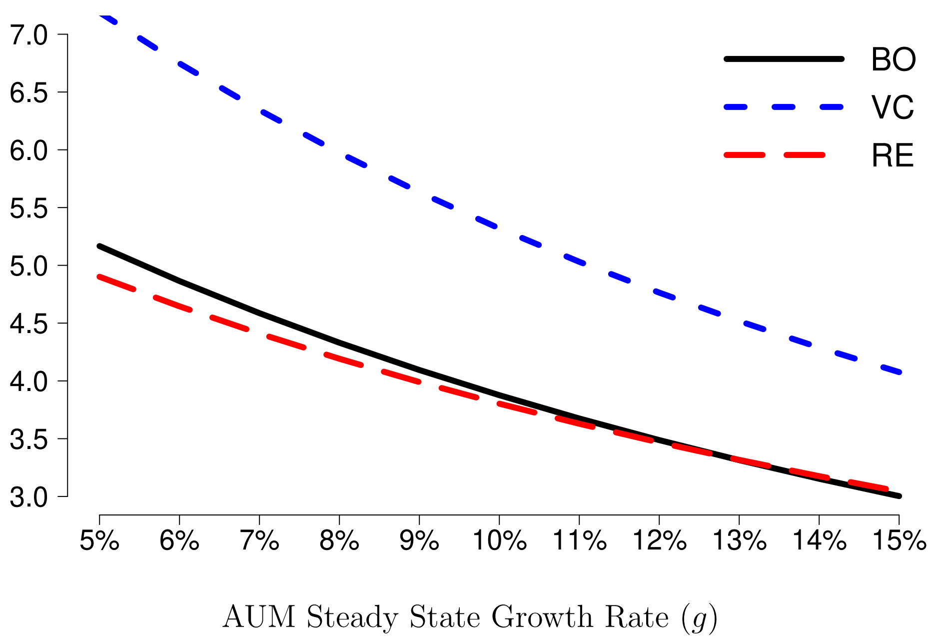


Figure 3
Private Capital Allocation Multiplier

This figure details our process for calibrating the allocation multiplier of each private capital asset class: buyout (BO), venture capital (VC), and real estate (RE). Panel (a) plots the total NAV of funds in each asset class relative to their size (i.e., total commitment) averaged over time, with the x-axis reflecting the time since the fund's inception year (i.e., the figure displays $\mathrm{NAVf}^{(h)}$ from Equation 14 for a strategy in which I_v includes all funds with vintage year v). Panel (b) displays, for each asset class, λ as a function of g based on Equation 14 by combining the $\mathrm{NAVf}^{(h)}$ estimates in panel (a) with different g values. Subsection 2.2 provides data details and Subsection 2.3.4 provides a description of the results in this figure.

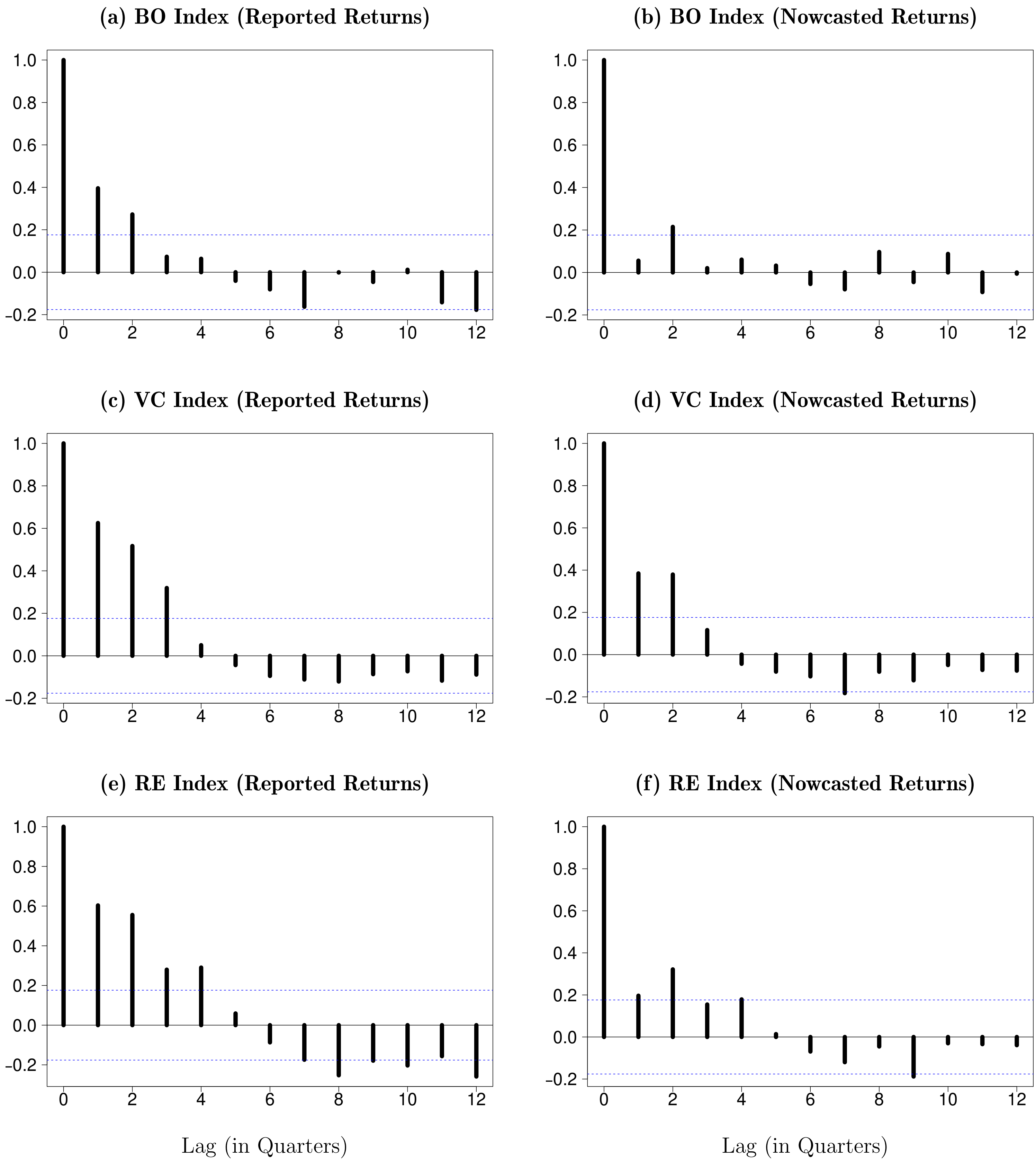


Figure 4
Return Autocorrelations of Aggregate Private Capital Portfolios

This figure plots the return autocorrelations for value-weighted private market indices. The private market indices cover buyout (BO), venture capital (VC), and real estate (RE). The returns on the private market indices are obtained using reported NAVs (Panels (a), (c), and (e)) and nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing (Panels (b), (d), and (f)). Subsection 2.2 provides data details and Subsection 3.2 provides a description of the results in this figure.

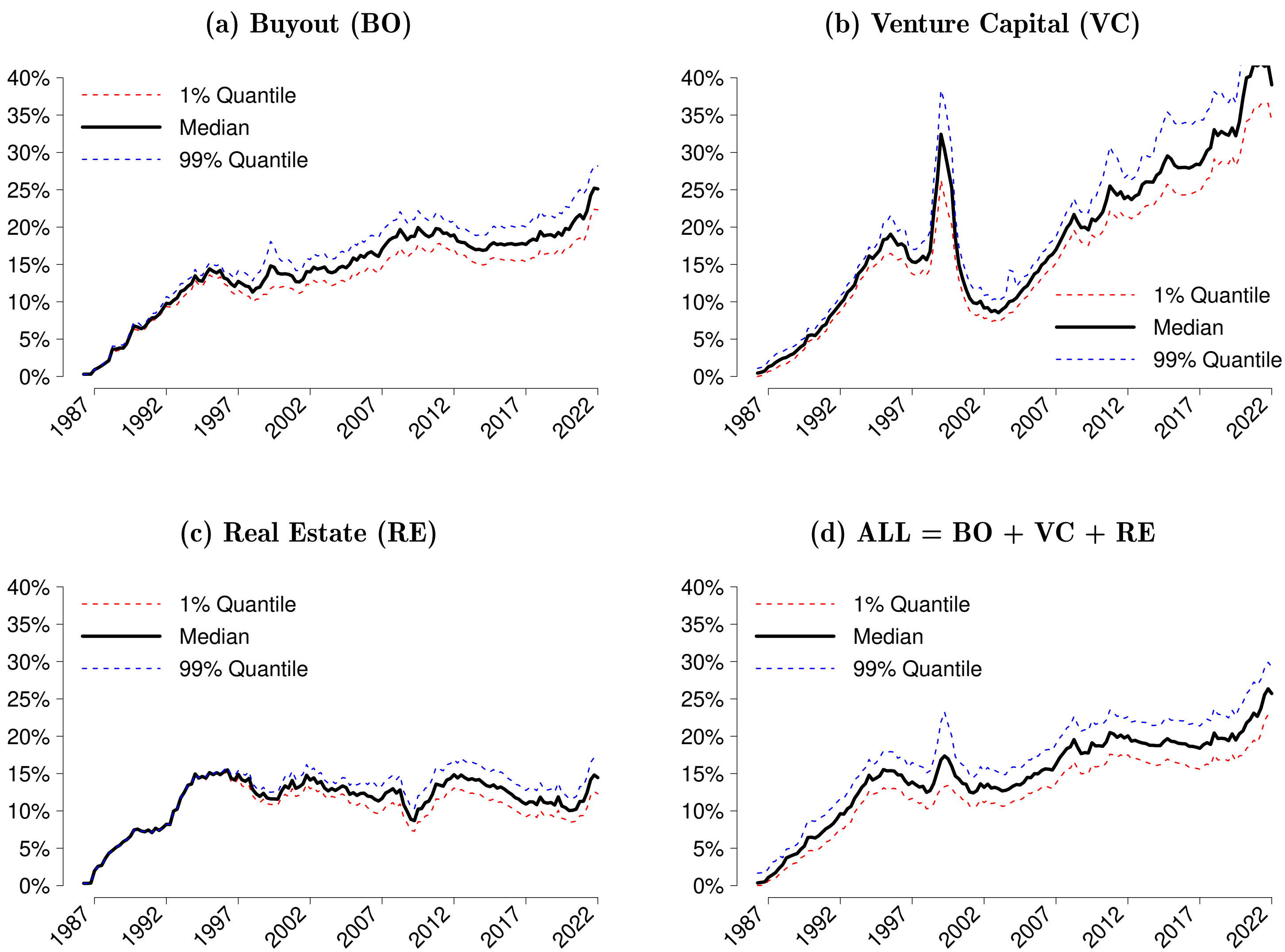
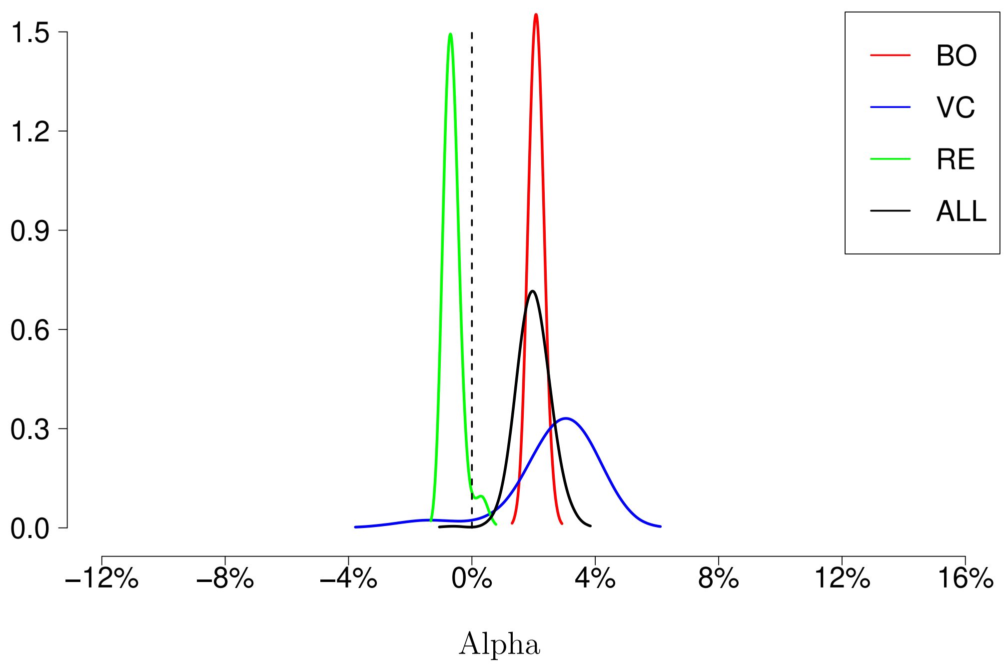


Figure 5
Private Capital Allocation Distribution

This figure plots the time-series of private capital weights (w_t) in the simulations for each private capital asset class: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The black line provides the median w_t while the blue and red lines provide the w_t at quantiles 1% and 99% of the w_t distribution each period. In all cases, the contribution strategy targets 20% w_t at steady state but the actual w_t oscillates over time. The first few years reflect the ramp-up period, with the private capital allocation starting at 0% and increasing towards 20%. Later periods reflect large oscillations due to the inability of the LP to control GP's capital calls and distributions. Subsection 2.2 provides data details and Subsection 3.3 provides a description of the results in this figure.

(a) Diversification



(b) Diversification+Sampling

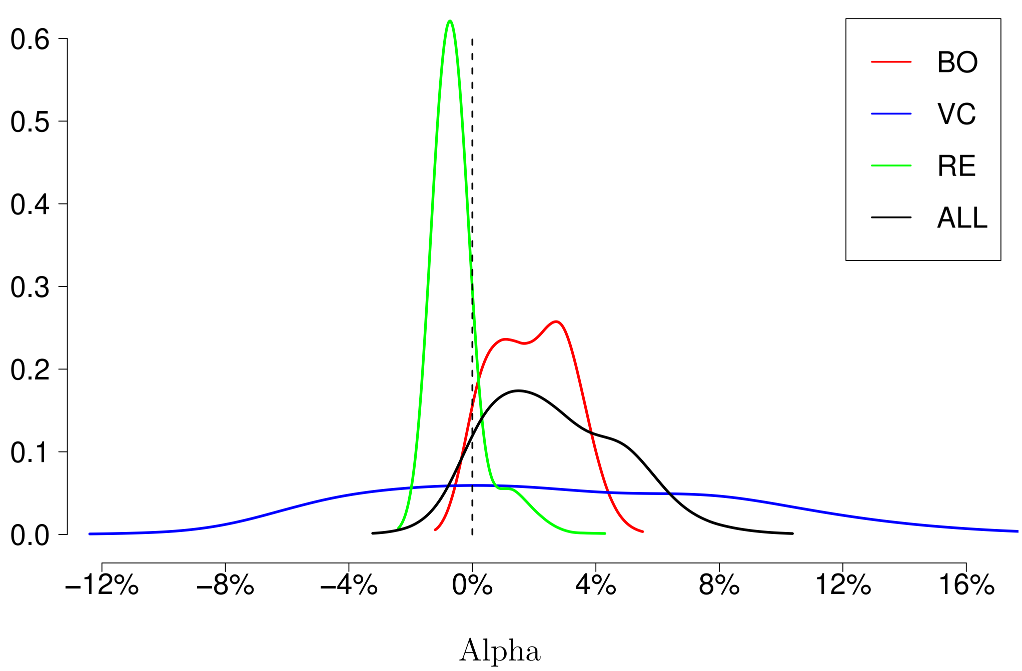
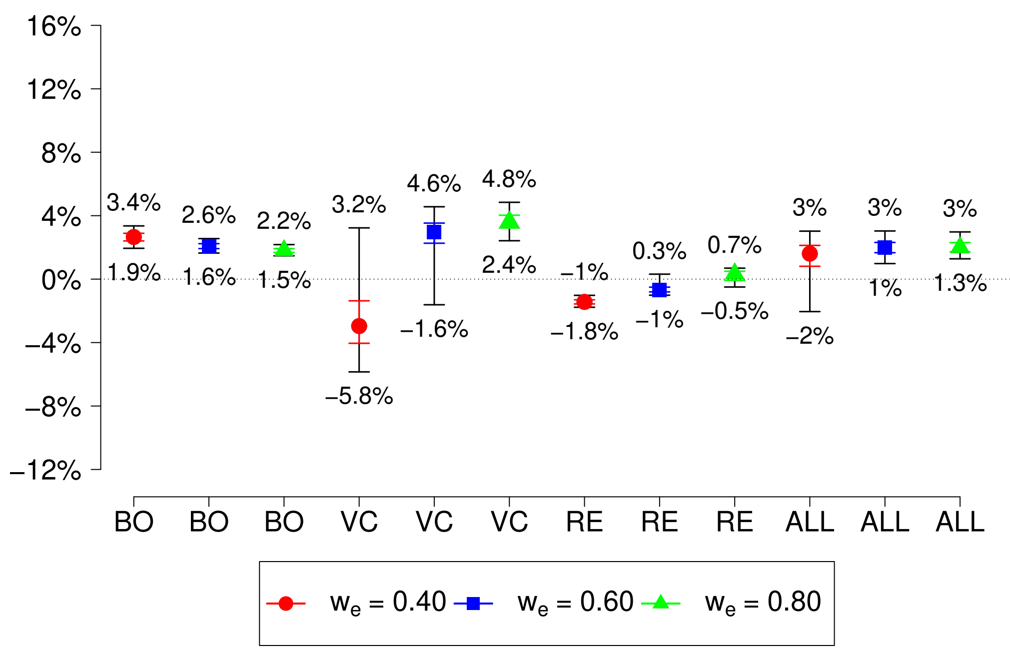


Figure 6
Private Capital Alpha: Smoothed Density Functions

This figure plots the smoothed density functions for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details on these simulations). All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 3.3 provides a description of the results in this table.

(a) **Alpha 95% CI: Diversification**



(b) **Alpha 95% CI: Diversification+Sampling**

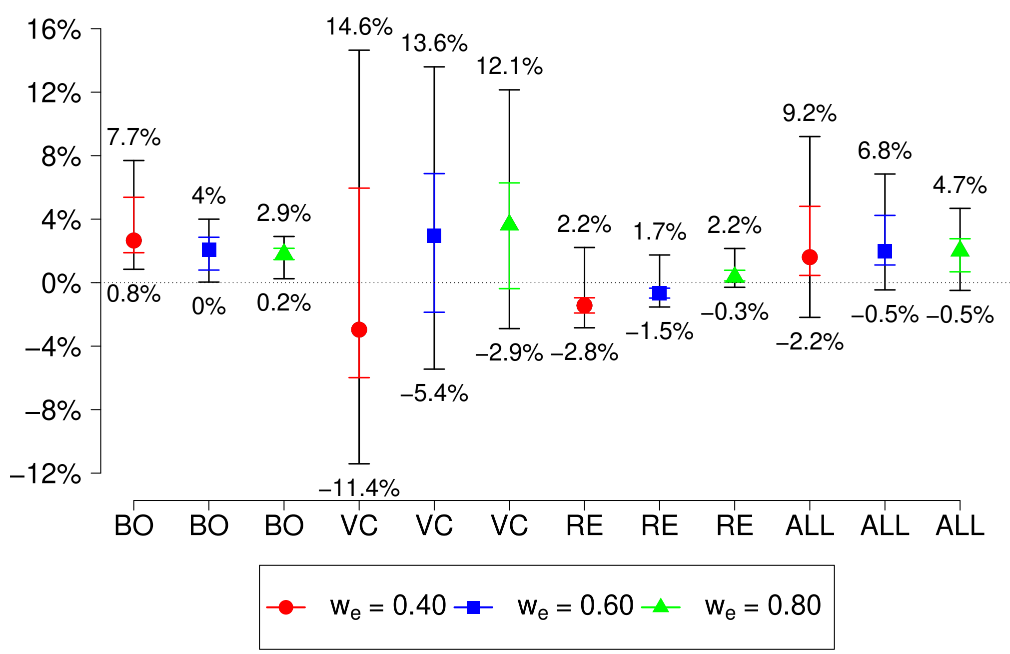


Figure 7
Alpha 95% Confidence Intervals: Alternative Public Market Portfolios

This figure plots the 50% and 95% confidence intervals for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details). The dots reflect the median alpha values across simulations for the given specification. The blue bars are for our baseline specification whereas the red and green bars are for specifications with alternative public equity allocations. All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 4.1 provides a description of the results in this table.

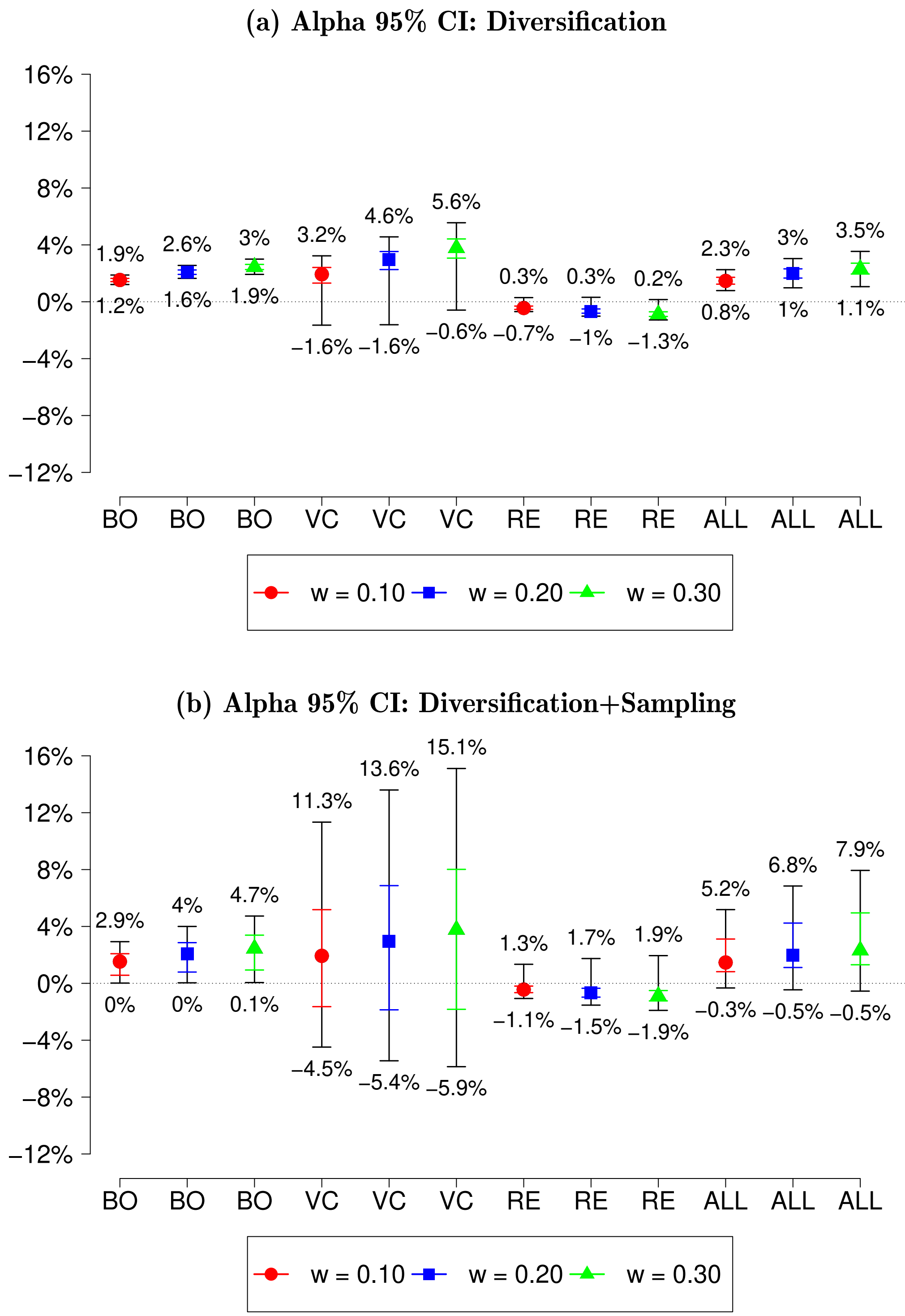


Figure 8
Alpha 95% Confidence Intervals: Alternative Target Allocations to Private Capital

This figure plots the 50% and 95% confidence intervals for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details). The dots reflect the median alpha values across simulations for the given specification. The blue bars are for our baseline specification whereas the red and green bars are for specifications with alternative private capital allocations. All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 4.2 provides a description of the results in this table.

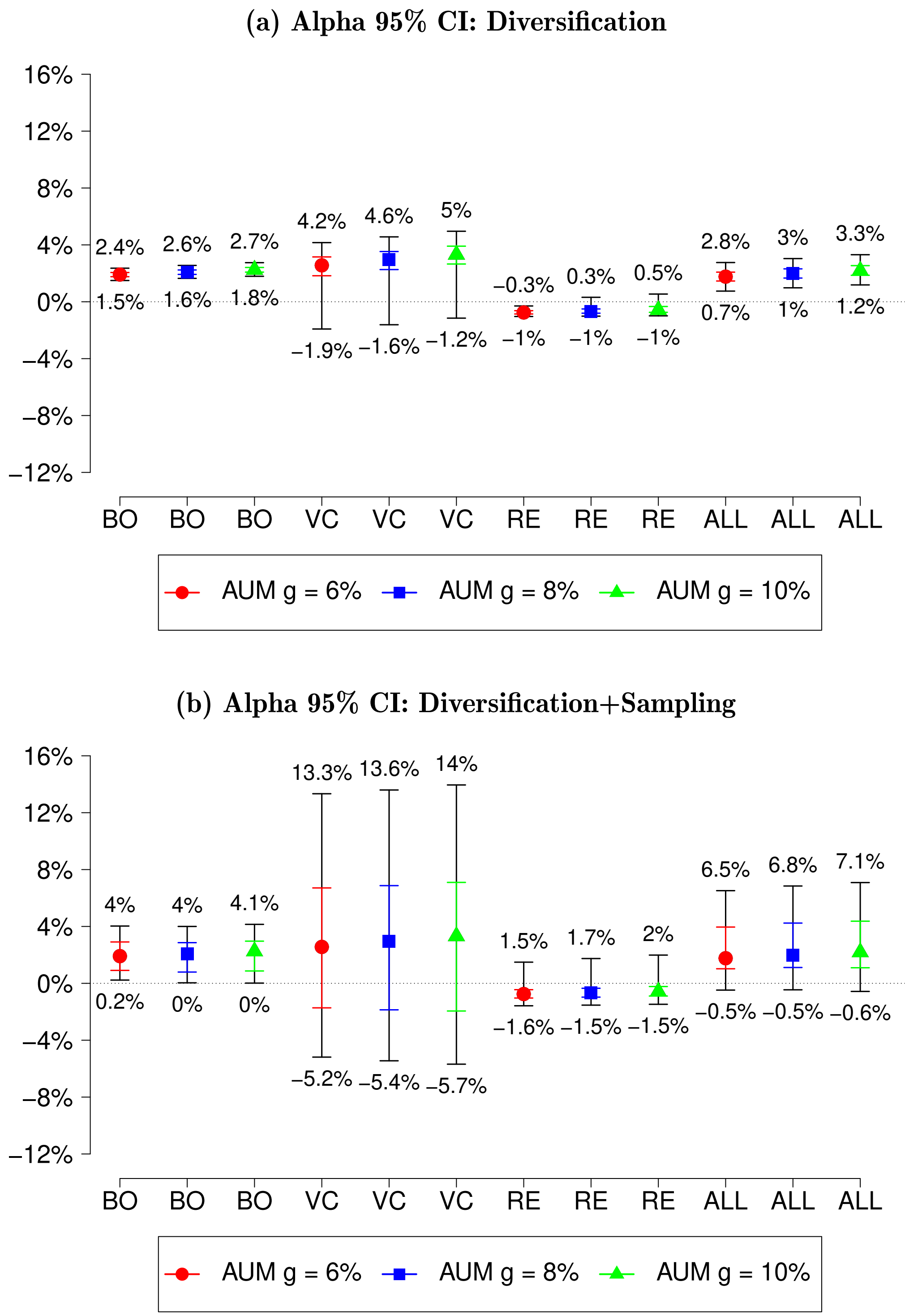
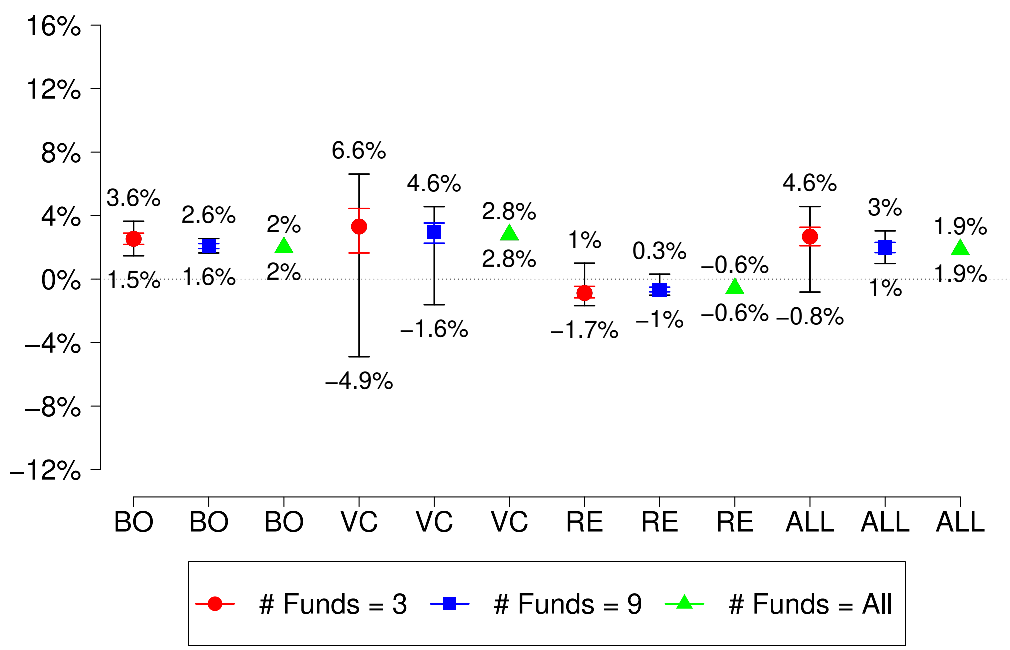


Figure 9
Alpha 95% Confidence Intervals: Alternative AUM Growth for Commitment Calibration

This figure plots the 50% and 95% confidence intervals for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details). The dots reflect the median alpha values across simulations for the given specification. The blue bars are for our baseline specification whereas the red and green bars are for specifications with alternative AUM growth. All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 4.3 provides a description of the results in this table.

(a) Alpha 95% CI: Diversification



(b) Alpha 95% CI: Diversification+Sampling

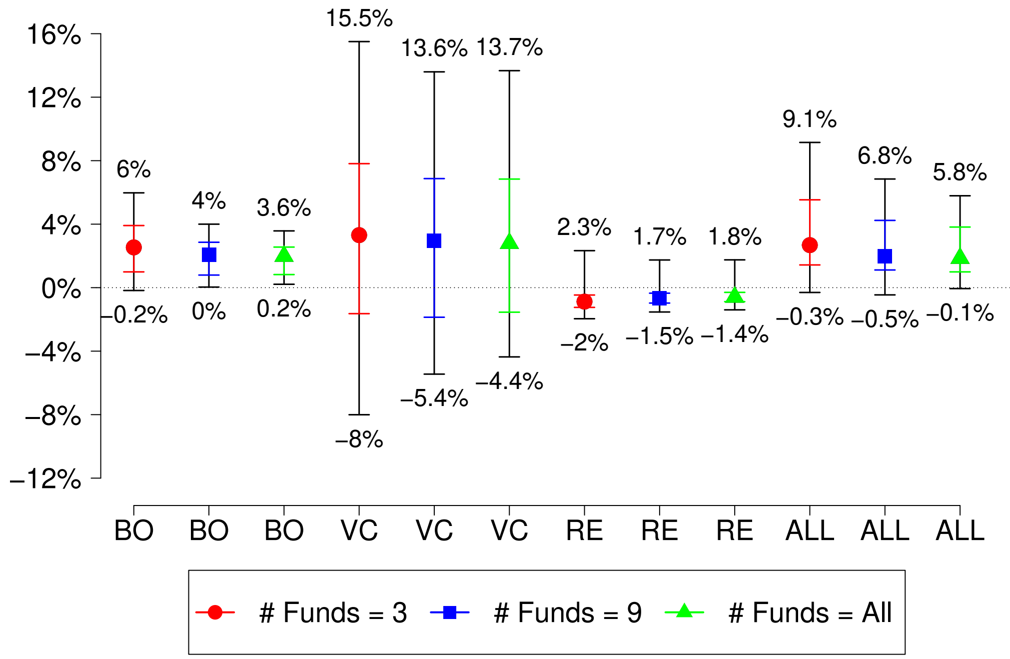
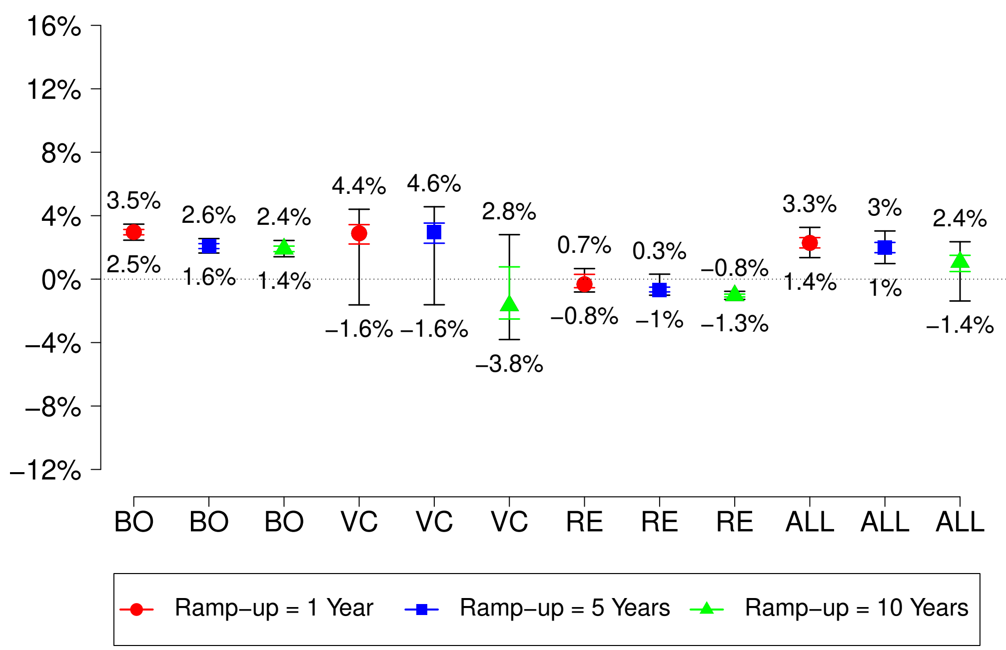


Figure 10
Alpha 95% Confidence Intervals: Alternative Levels of Private Capital Diversification

This figure plots the 50% and 95% confidence intervals for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details). The dots reflect the median alpha values across simulations for the given specification. The blue bars are for our baseline specification whereas the red and green bars are for specifications with alternative levels of diversification. All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 4.4 provides a description of the results in this table.

(a) Alpha 95% CI: Diversification



(b) Alpha 95% CI: Diversification+Sampling

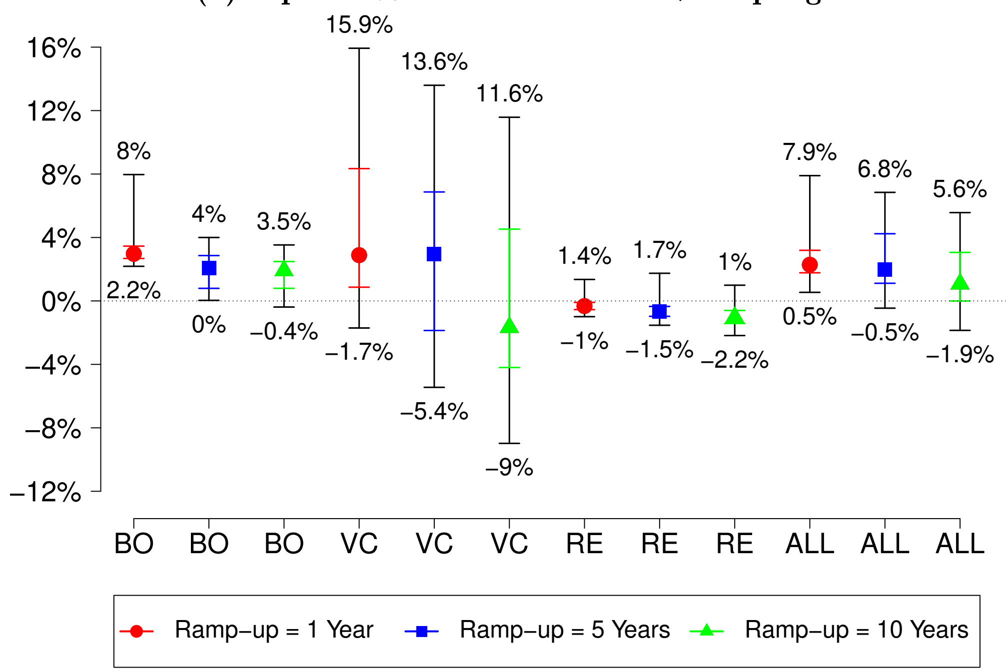
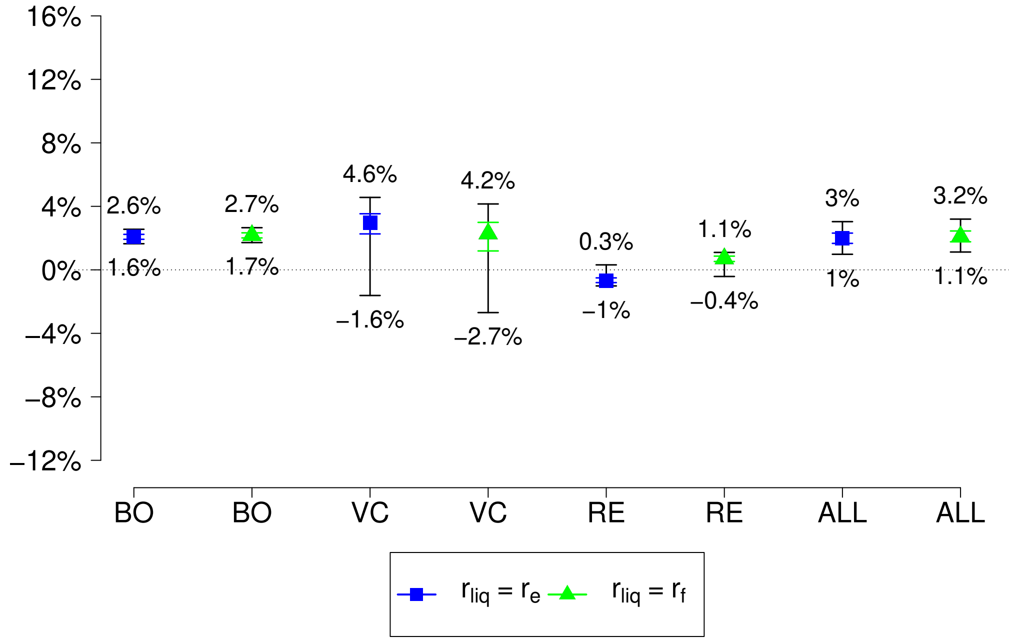


Figure 11
Alpha 95% Confidence Intervals: Alternative ramp-up Periods

This figure plots the 50% and 95% confidence intervals for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details). The dots reflect the median alpha values across simulations for the given specification. The blue bars are for our baseline specification whereas the red and green bars are for specifications with alternative ramp-up periods. All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 4.5 provides a description of the results in this table.

(a) Alpha 95% CI: Diversification



(b) Alpha 95% CI: Diversification+Sampling

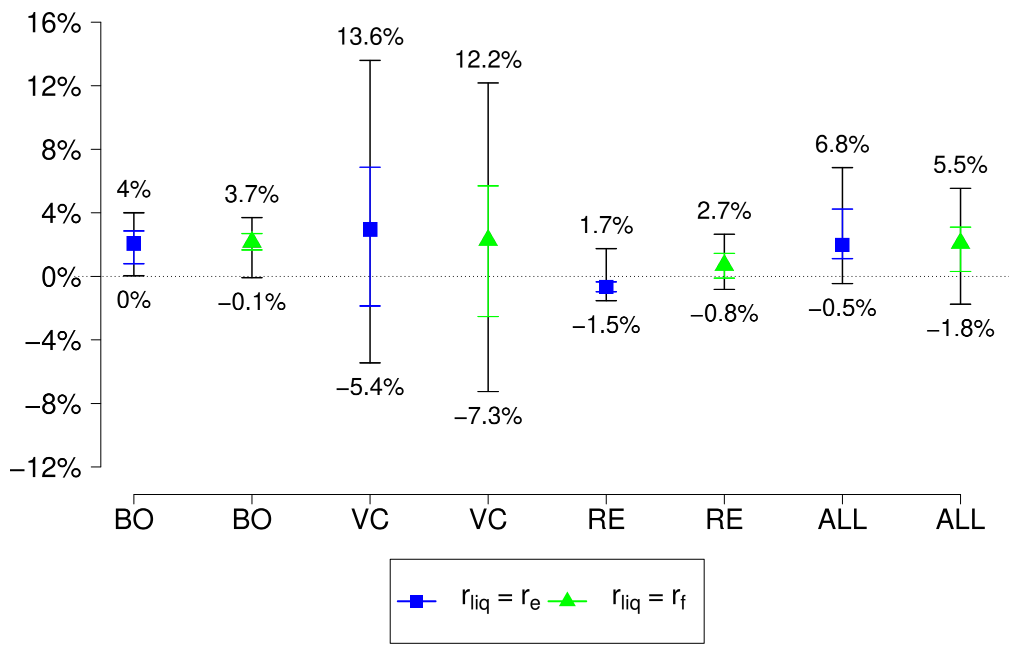


Figure 12
Alpha 95% Confidence Intervals: Alternative Liquid Asset

This figure plots the 50% and 95% confidence intervals for the alpha of each private capital asset class we study: buyout (BO), venture capital (VC), real estate (RE), and the three combined in a value-weighted manner (ALL). The results are based on 5,000 simulations that account for important considerations related to underdiversification, illiquidity, and LP portfolio allocations (see Section 2 for details). The dots reflect the median alpha values across simulations for the given specification. The blue bars are for our baseline specification whereas the green bars are for an alternative specification with Treasury bills as the liquid asset. All private capital returns use nowcasted NAVs from Brown, Gredil, and Kaplan (2019) to adjust for NAV smoothing. Panel (a) accounts for the uncertainty associated with an LP that commits capital to only 9 funds each period. Panel (b) compounds the uncertainty associated with underdiversification and sampling variation. Subsection 2.2 provides data details and Subsection 4.6 provides a description of the results in this table.

Internet Appendix

“The Private Capital Alpha”

By Gregory Brown, Andrei S. Gonçalves, and Wendy Hu

This Internet Appendix is organized as follows. Section A contains technical derivations required to support the results in the paper.

A Technical Derivations

This section derives Equation 2 in the main text.

Equation 6.6.17 in Campbell, Lo, and MacKinlay (1997) (originally derived in Gibbons, Ross, and Shanken (1989)) shows that the the factor model

$$r_t^* \ = \alpha^* \ + \ \beta \cdot r_{p,t}^* \ + \ \epsilon_t \tag{IA.1}$$

implies

$$\frac{\alpha^{*2}}{\sigma^2[\epsilon_t]} \ = SR[r_{pp,t}^*]^2 \ - \ SR[r_{p,t}^*]^2 \tag{IA.2}$$

where $r_{pp,t}^* = w^* \cdot r_t^* + (1 - w^*) \cdot r_{p,t}^*$ represents the ex-post maximum Sharpe ratio portfolio that can be formed with $r_{p,t}^*$ and r_t^*.

Noting that the coefficient of determination of the factor model in Equation IA.1 is given by $Cor[r_t^*, r_{p,t}^*]^2 = 1 - \sigma^2[\epsilon_t]/\sigma^2[r_t^*]$, we have

$$\sigma^2[\epsilon_t] \ = \sigma^2[r_t^*] \cdot \left(1 - Cor[r_t^*, r_{p,t}^*]^2\right) \tag{IA.3}$$

which we can substitute in Equation IA.2 to get

$$|\alpha^*| \ = \sigma[r_t^*] \cdot \sqrt{\left(1 - Cor[r_t^*, r_{p,t}^*]^2\right) \cdot \left(SR[r_{pp,t}^*]^2 \ - \ SR[r_{p,t}^*]^2\right)} \tag{IA.4}$$

Now, note that the a positive (negative) α^* implies a positive (negative) w^* and thus Equation IA.4 simplifies to

$$\alpha^* \ = sign(w^*) \cdot \sigma[r_t^*] \cdot \sqrt{\left(1 - Cor[r_t^*, r_{p,t}^*]^2\right) \cdot \left(SR[r_{pp,t}^*]^2 \ - \ SR[r_{p,t}^*]^2\right)} \tag{IA.5}$$

Finally, since $r_{pp,t}^*$ is the ex-post maximum Sharpe ratio portfolio that can be formed with $r_{p,t}^*$ and r_t^*, we have that $SR[r_{pp,t}^*]^2 \ \geq \ SR[r_{p,t}^*]^2$, and thus $SR[r_{pp,t}^*]^2 - SR[r_{p,t}^*]^2 = \left|SR[r_{pp,t}^*]^2 - SR[r_{p,t}^*]^2\right|$ so that Equation IA.5 can be written as

$$\alpha^* \ = sign(w^*) \cdot \sigma[r_t^*] \cdot \sqrt{\left(1 - Cor[r_t^*, r_{p,t}^*]^2\right) \cdot \left|SR[r_{pp,t}^*]^2 \ - \ SR[r_{p,t}^*]^2\right|} \tag{IA.6}$$

which is Equation 2 in the main text.

References for Internet Appendix

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton University Press.

Gibbons, M. R., S. A. Ross, and J. Shanken (1989). "A Test of the Efficiency of a Given Portfolio". In: *Econometrica* 57.5, pp. 1121–1152.